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BEAUTY, THE IDEAL—OF THE ARTS, AND THE SCIENCES*

BY

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PURPOSE IN THE UNIVERSE

Many in the past have wondered as to the reason for all they saw around them. Are we to suppose that all things, ourselves included, are the results of the operations of chance, or is there something more?

Fortunately, for what I have to say this evening, it has not been necessary to await the answer to this great question, with its various subquestions. It is not even necessary to decide whether the question has meaning. Were it otherwise, I fear you would have no lecture this evening, at least from me; and, if our President and Dr. Allen had arrived expecting to find me all primed for an address, the best they might have hoped for would have been to find a person hiding in a back room, with all the evidence of mental derangement, wishing that he had something to say but in a lamentable condition in which, the more he thought about his speech the less confidence did he have in giving it, or, in fact, in saying anything whatever.

In order to find courage to address you at all, it has been necessary for me to seek some starting point—something to which I can pin my faith and which can form a basis of what I have to say. I must avoid contaminating this starting point with other matters which surround it, which claim relationship to it and which clamor for expression. Of these various contestants, which, as it were, hold conclave in my soul and which cry out to be heard, I must act as chairman, and I must keep the meeting locked to the purpose I have in mind.

And so, as my first firm point of departure, I make recognition of the

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fact that the universe has structure. It has design. I care not why it has design or whether any being in the form of a person designed it. That design exists is evident from all I see around me. It is evident in the intriguing harmony of nature, in the clever way in which the laws governing the universe conspire to produce the maximum of richness of content from the minimum of fundamentals at the basis of all that content. The universe stands to the engineer and the artist alike as an ideal of achievement in their respective domains, so that the cleverness of that which they see around them is something which, even if it were far less than what it is, would have invited their envy but which, in actuality, is beyond envy and to be weighed only in terms of admiration amounting to awe. And with this realization is born my confession of a faith. And the faith lies in the belief that the design contains within itself what I may call a purpose—a purpose which provides for the existence of something which I may regard as good. I must rule as out of order any demand of some member of the conclave to which I have referred to the effect that he would like me to discuss the question of *whose* purpose is involved. I am content that this situation which, for want of better words, I have termed the existence of a purpose, is in fact a reality. I care not whether you object to the word purpose. If you do, I am content with any other word. All that matters is the potentiality of providing for the existence of something of which I may say: "It is good that this thing exists," something which, without the design of the universe around me, would not have existed at all. Let me illustrate what I mean.

Suppose there had been nothing. Then there certainly would have been no unhappiness, but there would also have been no happiness. As far as inanimate nature is concerned, it would seem that there would be no loss had there been no universe at all. I do not think that the planets would mind if a decree went out that they should no longer exist. However, directly I think of living things as part of the structure, the picture is different, for then I have the potentiality for happiness, a *positive* thing which has no meaning except as a counterpart of life. I have, alas, also the potentiality for unhappiness—again a thing with no meaning apart from living things.

HAPPINESS THE PURPOSE

For man, there are three states of happiness. First there is the state of what I may call negative happiness, ordinarily called unhappiness. This is the state which man himself is continually striving to destroy by the elimination of disease, wars, and other catastrophes of analogous kinds. Then there is the state of zero happiness in which a man is neither happy nor unhappy. This is the state of a corpse. If I may risk a paradox in words, but a paradox which perhaps conveys some meaning, I might say that the state of zero happiness in man is something like the state of the universe if there were no universe, or if

the universe, though existing, had no interesting potentialities. My friends, the specialists in nuclear physics, would call this the ground state of the universe, the state of maximum deadness, the state possessing no potentialities of its own, so that that which is to be found in this state must rely on external agencies to convert it to some other state, where its existence is accompanied by greater possibilities. Then there is the third state of happiness, the state of positive happiness, in which a man alive is happier than if he were dead or had never been born. This is the state which man, by his efforts, is continually endeavoring to enhance at the expense of the states of zero happiness and negative happiness; and, the very fact that man has this urge, man who himself is part of the universe, suggests that such an ambition is a consequence of the design of the universe itself. Whether you designate that goal to which man, through his efforts, is progressing by the word "good" or by some other word, I care not. In terms of the ideas of man as he has developed, the goal is, by the very meaning of the word, good.

That man, therefore, seeks to progress and indeed is progressing to a more complete state of happiness is evidenced in the facts around us. But little faith is required to believe that the fundamental purpose of the universe is to provide for the situation where happiness and beings to experience happiness can exist. Without such a faith, I can but believe that the whole universe is a blunder, and all nature a joke.

If I may venture the presumption, I think I may say that were I gifted with the powers of a creator who could bring into existence a universe out of nothing, I could wish for no more satisfying achievement than that of creating something in which happiness had the potentiality of existing.

BEAUTY, THE INSPIRER OF HAPPINESS

And what are the elements which determine happiness? Happiness is something which does not exist in uniform continuity. It usually takes the form of a sequence of moments of exaltation, moments which, through memory, leaven with a sense of well-being all the periods of time which exist between the moments of exaltation. These moments of exaltation are born of sudden realizations of certain elements of what I shall characterize as "beauty." I take this word to cover a vast domain of attributes of which, in what follows, I shall endeavor to trace examples. To express these elements, to bring them into existence, or to unveil them as they already exist constitutes the whole worth-while purpose of living beings.

On the screen[2] is a flower. Why on earth did nature provide that delicate green border? I can almost see an intent to add a last touch of beauty before regarding the creation as complete.

[2] The pictures mentioned in the lecture have not, in all cases, been reproduced in the JOURNAL, since the text is fully explanatory.

There may be some who doubt my emphasis on this matter and who would give greater weight to what they might term the practical attributes of our civilization. In this connection I have sometimes meditated on a supposed conversation between a rabid materialist and an artist of the "Art for Art's sake" type. The discussion takes the form of an appraisal of the value of the pictures which Michaelangelo painted upon the ceiling of the Sistine Chapel in the Vatican.

"Of what use are those pictures?" asks the utilitarianist. "They do nobody any good and only wasted the time of Michaelangelo who painted them."

"And what kind of creative work would you regard as of use?" asks the artist.

"Well, the development of the steam engine or the automobile," says the utilitarianist.

"But why are these of use?" says the artist.

"Because they enable one to move about faster and get more done," says the utilitarianist.

"But why move about faster and get more done?" says the artist.

"Because by doing so you create wealth for yourself and others; you save time and are enabled to enjoy more leisure," is the rejoinder.

"And what is the use of money and leisure?" asks the artist. "Is it not rather boresome to have nothing to do?"

"Oh, it is not necessary to do nothing," is the reply. "You can travel and enlarge your mind."

"But," says the artist, "what is the good of traveling? You only get seasick and very tired."

"Oh," replies the utilitarianist, "it is a wonderful experience to travel. You can go to the Old World and visit all those places of classic renown: Paris, Venice, London."

"But," says the artist, "is that not very disturbing? I hear that many of these places are unsanitary. The food is not what you are accustomed to, and sometimes the people are not over friendly."

"Oh, those are but small matters," says the utilitarianist, "they are far outweighed by all of the riches you fall heir to. You can bask in the exhilarating sun of the Alps. You can drink in the beauties of the Mediterranean. You can visit ancient Rome; and by the way, when you are in Rome, do not fail to see those marvelous pictures which Michaelangelo has painted in the Sistine Chapel of the Vatican."

And so I have wondered if we should be far from the truth if we should maintain the thesis that the only ultimate excuse for the existence of the things utilitarian is that they provide the means whereby we may enjoy the things non-utilitarian.

And now to return to my main theme, what are those elements which stimulate the mind to exaltation? Of what nature is this thing which animates the living world, this phantom which all men seek though oft unconscious of the seeking, this child of nature to provide

for whose birth perhaps the universe was created, this thing so undefined in its totality yet so meaningful in its individual revelations, this sprite which takes so many forms on the stage of life, this jewel with so many facets in music, in poetry, in sculpture, in painting, in science, this thing which adorns even the material creations of our civilization—this beauty?

THE NATURE OF BEAUTY—BEAUTY IN SCIENCE

As one peers into the realms of those things which exalt the mind and seeks for the stimuli which are responsible for the exaltation, he finds, in the last analysis, no gargantuan pictures, but rather pictures of the opposite kind. He finds great simplicity. The conscious or unconscious recognition of simplicity as the forebear of grandeur is one of the strongest elements of appeal to the imagination.

On the screen, you see the motions of the planets as seen from the earth. This is Neptune; this is Uranus; this is Saturn; this is Mars. The motions are complicated, difficult to understand, difficult to harmonize with anything. The endeavor to breathe some meaning into these motions defied the ingenuity of the astronomers and philosophers of old. However, when we look at these motions as seen from the sun, rather than from the earth, we get the picture which is now before you. And if we look at the motions of the individual planets, we see the following.[3] Now, all is harmony; the planets move in circles or ellipses around the sun as focus. This was the discovery of Copernicus; and, the realization that simplicity could be dissected out of complexity in this way or, if you like, that one could find the father of complexity in simplicity, is a thing which animates the imagination of the mathematician and enlivens him with a sense of exaltation so that, spontaneously, and without urge from the outside, he will declare that this concept is indeed beautiful.

But the motions of the planets provide further illustrations. Even though a great gain in simplicity had been realized in recognizing the harmony of planetary motion as viewed from the sun, there yet remained the problem of a further harmonization to account for the remaining properties of the motions, one property, for example, being represented by the fact that a line joining the sun to a planet sweeps out equal areas in equal times, the planet moving faster when it is near to the sun than when it is more remote. It was Newton who saw that this additional feature and many other features could be harmonized under one simple controlling principle, a principle called the law of gravitation, a principle to the effect that each planet moves in such a manner that that part of its acceleration which is directly towards the sun varies inversely as the square of its distance from that body. In this relatively simple statement of principle with relatively minor extensions is harmonized all the story of classical astronomy. Through this principle one can

[3] Here followed illustrations of circular and elliptic orbits in the form of a moving picture.

calculate the history of the heavens from the remote past to the distant future. With incredible accuracy one can predict the time of occurrence of eclipses, the time of return of comets, and a hundred other things. Even the story of the tides eventually yielded its secrets to the implications of this simple law. Here again, we have an example of a thing which is beautiful to the man of science. If there be any students in the audience who are finding difficulties in their studies of physics or mathematics, they will doubtless concur with malicious cynical satisfaction in the statement which I now make, to the effect that the function of the man of science is to take a simple thing and make it complicated; and yet, I can stand firmly behind this statement in realizing that the complexity which science breeds from simplicity is a complexity which mirrors the grandeur of the universe.

The man of science has passed through periods in which he has strayed from the ideal in the appreciation of the beauty of complexity as born from simplicity, and has sought what he may have deemed more practical satisfaction in the worship of a false god whose name is "understanding." The great impetus given to science by the discoveries of the Newtonian era and their extensions in the fields of dynamics and hydrodynamics to a correlation of the motions of matter in bulk, even the internal motions as exemplified in acoustical vibrations, invited mankind to a certain illusion of satisfaction in the belief that he was on the verge of a materialistic understanding of nature. He hoped to see all new things which, at the moment, he did not understand, mirrored so closely in analogies to the material things which he felt he did understand, that he could form a bridge of understanding from the latter to the former and so rest in contentment that now he understood everything. Alas, careful analysis of the situation shows that his original confidence of understanding of the things which were familiar to him was in each individual born of a period in his life in which he was too young to be critical and accepted as well understood the things which were familiar. At this stage he was too young to question the validity of his understanding of these familiar things; and later, when he became more critical in the analysis of new information, he omitted to extend his new sophistication in philosophy to the realm of his old experiences because time had made these old experiences so familiar to him that they seemed to call for no further thought or analysis. The acceptance of these old things represented the spirit of the common sense of his day; and, if you questioned him too rigidly, his new-born critical conscience might, for a moment, challenge his older complacency, so that, to escape your arguments, he would be apt to make that final personal appeal to his own anatomy in assuring you that he felt in his very bones that such and such was so and beyond his bones you would not get.

The bones, and common sense, formed a sort of unholy alliance in the mid-Victorian era, an alliance which sought to cement, on a firm basis, what I may term the materialistic ideal in natural philosophy,

an ideal which lived in the belief that although all matters were not at the moment correlatable in that philosophy, some day they would be. The alliance between common sense and the deep-seated bones was supported by powerful influence and resisted dislodgment. However, when new concepts born of the theory of relativity forced upon us a recognition of the fact that there was much in our complacent belief as to the nature of space and time which was illusory, much which was indeed nonsensical, many were driven to search their conscience in philosophy. Then appeared strange new phenomena having to do with atomic physics which resisted amalgamation with the older schemes of thought, and even the old science of classical astronomy became clothed in new raiment born of the general theory of relativity. Clad in this new raiment, the old astronomy lost not the spirit of mathematical simplicity brought to light in the Newtonian era. Indeed, the beauty of the mathematical design was enhanced through greater generality to a new beauty which transcended even that of the Newtonian concept. It was not mathematical form and design which suffered by the requirements of the new more sophisticated philosophy, but only the materialistic picture, the false god who had claimed to be the origin of everything and who failed when things foreign to his nature were demanded of him.

And so today we have, in the modern theory of atomic and nuclear structure, very different ambitions of understanding than those prevailing a century ago, but structure and mathematical design remain in enhanced form, and the beauty of the design has lost nothing in the advance of years but has, indeed, gained in combining the freshness of a new youth with the experience of maturity. No longer do we seek satisfaction in an illusive concept of reality in the universe, but rather in the beauty of the design itself and in the richness of the phenomena of our experiences which it harmonizes. Our present theory of nature, in contrast to an abortive attempt to see all that is around us in the workings of huge machines, recognizes rather a picture something like an oriental rug; and, as such a rug, in the minds of those who constructed it, symbolizes certain beliefs and experiences in the design, so, in its formal relationship, our picture symbolizes, and knits together, those happenings of all kinds which it is the function of the natural philosopher to study. And so the Grand Architect of the universe becomes the idealization of an artist rather than an engineer, as that term is customarily understood. To the mathematician, the beauty of the whole structure is to be seen in the fact that the relatively few controlling fundamentals of the design interweave in such manner as to provide that marvelous richness of potentiality evident to our experience, a richness which embraces and correlates our understanding of the very birth of our universe some thousand million years ago, its growth through adolescence to maturity in which life appeared and sought its own development, all being anticipated, as it were, in the provisions of the funda-

mental laws which themselves were evident in the eons of the past, when little which we now behold was in evidence.

SPONTANEOUS APPEAL, VERSUS APPEAL FOUNDED UPON TECHNICAL KNOWLEDGE

Turning to a less general concept of beauty as represented by mathematical design, it is of interest to recognize that, in general, beauty may be divided into two types, that which presents an immediate appeal to him who beholds it, and that which emerges only in the light of understanding. Sometimes beauty presents itself in one form, sometimes in the other, and sometimes it embraces both.

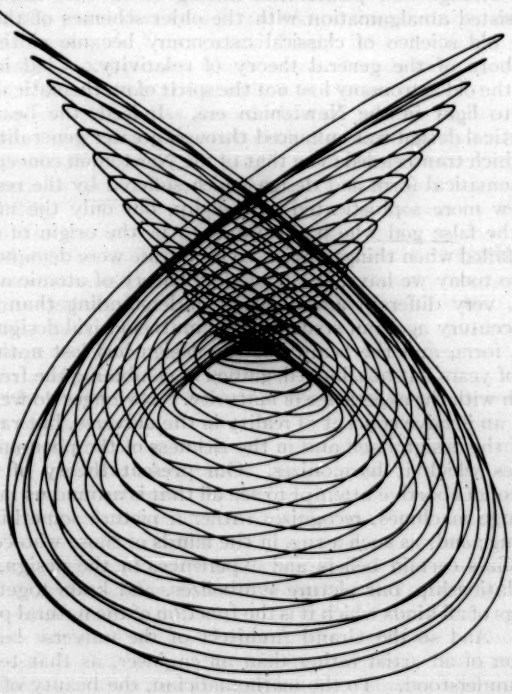


FIG. 1.

On the screen (Fig. 1) is a geometrical design. I think that many will spontaneously appreciate it as a thing possessing certain elements of beauty, and its inspection brings forth a pleasurable response. We

have, in fact, an example of beauty in its spontaneous appeal. Let me show you, however, how this design was produced.

I have here (Fig. 2) a pendulum which can swing back and forth in a plane parallel to the lecture table. It carries a board to which is attached a piece of paper on which a design may be made. I have also

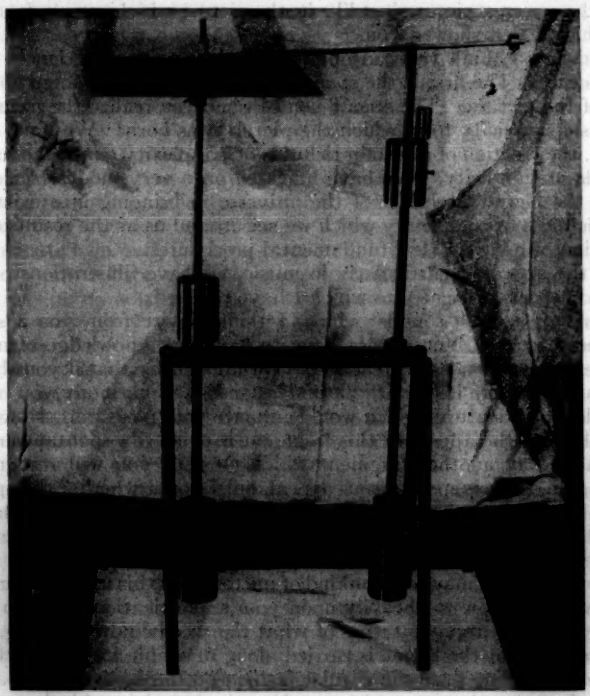


FIG. 2.

another pendulum which swings in a plane perpendicular to the lecture table and which carries a pen capable of writing on the table. If the pendulum carrying the pen moves while the other pendulum is stationary, a straight line will be drawn perpendicular to the lecture table. Here it is. If the pendulum carrying the paper moves while the other pendulum is stationary a straight line parallel to the edge of the table will be drawn. Here it is.

If I now allow both pendulums to move, that which will be drawn will have a more complicated form. The form will naturally depend upon the relation between the periods of vibration of the two pendulums. I have arranged the pendulums so that their periods of vibration are in the ratio of 3 to 2. After a short interval of operation we will examine the result. Here it is. For your greater ease in examining, I will throw upon the screen a picture just like it, the picture which was, in fact, the one which I first showed you.

And now I think you can obtain two pleasurable experiences from this picture. The first is the spontaneous reaction to the mere observance of the picture. The second comes when you realize the nature of the basic simplicity from which the picture was born. We have here, in fact, the creation of a certain richness of complexity out of the fundamentals of simplicity. We have imitated on a very modest scale the act of the Grand Architect of the universe in bringing into existence that richness of complexity which we see around us as the result of the operation of relatively few fundamental procedures.

In the arts, and particularly in music, we have illustrations of the twofold aspects of beauty to which I have referred.

Here are certain sounds[4] which, I think, invite from you a spontaneous reaction. You do not have to have any knowledge of music in order to experience this reaction. I will not deny that if you should look into the score itself with an understanding of harmony you might find additional features which would enhance your pleasure. However, by and large, the appeal of this fragment is primarily spontaneous.

I now play another fragment which most of you will recognize.[5] Here again, there is spontaneous appeal, but I doubt whether the appeal would be as universal as that of the *Marseillaise*. By the contemplation of certain structure, however, I can enhance the appeal. Here is a picture of the score.

Now I know it may seem unkind of me to throw this upon the screen, but I will not draw too heavily upon your sophistication. If you look at the top lines, representative of what the woodwind is playing, you will observe that the theme is carried along in simple fashion, with the big chords coming firmly in regular fashion on the beats of the bar, of which there are six to a measure. You, in fact, hear this:[6] On the other hand, if I call your attention to what the strings are playing, you will observe that the successive notes do not occur in such regular sequence but in a kind of syncopation with the wood wind. You hear, in fact, this, in which I have counted the individual beats to illustrate the relation of what is being played to the beats of the bar.

Once more I will play the first fragment, representative of what the

[4] Here was played a section from the *Marseillaise*.

[5] The first few bars of Brahms' First Symphony were played.

[6] Here was played the fragment in question, a tape recording of a piano being used.

woodwind is playing,[7] and I will follow it immediately by the second fragment, so that your ears will have in mind the first.—And now, while your ears are still attuned to these two fragments, I will play them together.—I think you will be conscious of a kind of strife between these two fragments and, to me at any rate, the consciousness of this strife provides a pleasurable experience which is superposed upon any spontaneous appeal which may have existed in the absence of a recognition of that strife.

I have chosen this very simple illustration because, indeed, it is simple, but I think most musicians will agree that throughout the whole rendering of a composition of this kind, situations analogous to what I have here depicted play their part in enhancing the beauties of the composition when their presence is recognized.

In this illustration I have taken an example perhaps intermediate between one of spontaneous appeal and one where the appeal is wholly in the recognition of the structure. As regards the latter, just as in mathematics, in golf, in chess, and in any other of the activities which exercise man's ingenuity, there arise conditions and problems whose solutions invite admiration and, by this token, take on a semblance of beauty, so in music, there is a kind of beauty which arises from the invention of new musical ideas, or from the manipulation of musical form in a manner intriguing to those who are engaged in similar activities. For the general layman to expect an appeal from such an achievement would be like expecting exaltation from the contemplation of a game of chess by one who is completely unversed in the principles of the game and is totally unacquainted with the moves. To such a person the game would present no elements of beauty at all, whereas, if that game were played in what would be deemed a masterful manner, as judged by an expert, it might indeed engender in that expert feelings which were quite akin to those which he associates with beauty in other realms of mental activity.

I should like to say something about this matter in relation to painting, but am conscious of my profound ignorance in this field. I might get along moderately well with the classics, but I have been informed by a noted artist that a certain picture which won an important prize in France was painted by a brush attached to the tail of a mule, and I will confess that I would not know how to attempt an analysis of this situation.

ORIGIN OF APPEAL IN MUSIC

To return to the question of the appeal inherent in such an art as music, and particularly to a discussion of those things which are associated with spontaneous appeal, it is of interest to inquire as to their

[7] The first fragment was played, followed by a space in which the lecture proceeded and then followed by the second fragment, after which again the lecture proceeded to the point at which the two were played together.

basic origin. In this connection, it must be recognized that the appeal under consideration is something experienced by human beings. Human beings are strange animals, and there is a great diversity of opinion among musicians as to what is good and what is bad. Indeed, becoming acutely conscious of this situation, I was, on one occasion, constrained to define music as "an art devised to provide a means by which its exponents may differ from one another as to its interpretation." Moreover, there is a kind of saturation principle applicable to mankind in continued exposure to things which all would, at first hearing, deem good. Indeed, an audience, and one of artists in particular, seems to possess an almost infinite capacity to become irritated. It learns to expect this irritation, and directly the performer appears it is on the look-out for trouble. This capacity to become irritated by frequent exposure to a good thing is not limited to music. I appear some day before you with a new tie which I have bought after much critical meditation. You say, "Swann has a nice tie." I visit you a week later and you say, "Swann is wearing the same tie; oh well, it is a very nice tie." After another week, I again appear before you in the same garb. You grumble to yourself, "I wonder why Swann always wears the same tie." After a few more visitations, you will say: "That tie of Swann's gets on my nerves; I can't stand the sight of it." If this situation goes on for a month or two, you will begin to acquire a subconscious feeling that I wear the tie to annoy you, and the tie will become a nucleus for the concentration and magnification of everything else you dislike about me, so that after a year you may be willing to murder me for rather ill-defined reasons, which really have their origin in that tie.

Now, in the case of a musical composition, we have two things, the written score which the composer has produced and which is the record of something in his brain, something which he believes will produce pleasurable emotional response and even exaltation in others. Then we have the technique of the conveyance of this message through the medium of sound. Both the message itself and the conveyance of it are restricted partly by principles which are matters of convention in the craftsmanship of composition and partly by other elements which can be characterized under the general designation "good taste."

ORIGIN OF RULES

Perhaps we can see better what is involved in these matters by appealing to an art much more ancient than music, the art embodying all that is involved in the use of language. Here the sensitivities even in the layman are more acutely developed than they are in music, for we have been speaking for 6,000 years and we have only been playing music of any appreciable complexity for 300 years. In language, the starting point is a story, or a poem, or something which has meaning in the abstract. In the conveyance of this abstract message through the medium of words certain conventions have evolved. As regards the

conventions themselves, they are not necessarily part of the message, and yet, if they are violated, I am offended. If somebody tells me a beautiful story and tells it in bad English, I may understand the story, but I am nauseated, so that my pleasure is destroyed. On the other hand, if the tricks of expression in the telling of that story are plentiful and governed by the conventions of the good taste of the day, the full meaning of the story is enhanced in my consciousness. As regards the formal conventions themselves, these have usually grown out of the past, and sometimes there seems no logical reason for their existence. Thus, for example, I can give no logical reason to explain why it should be bad to split the infinitive. I probably split it many times in my youth and only through constant fear of the frowns of the Emily Posts of literature was I brought to the stage of avoiding it. Then my conscience in the matter having been formed, I continued to avoid it because I came to dislike it, so that, in the end, I needed no frowns other than my own internal frowns to make me behave as I should in the matter.

Now it is to be recognized that in speech and in music conventions have grown gradually. That which we call harmony in music is a discipline which was not created overnight. Certain elements comprised under the designation "good taste" in music and in speech are things born one by one in the history of the past.

Often, particularly in youth, one senses an irritation on being bound too rigidly by rules of procedure and feels that his own individuality of expression is destroyed if he is bound too much by what he has been taught. And so, if he is very young, he lets what he feels to be his own individuality have full sway. He plays his instrument, or recites his poem. The result sounds marvelous to himself but abominable to everybody else. What is the explanation of this matter?

The fact is that everybody is born with certain feelings as to what is good and what is bad in expression and in composition. Some of these feelings are really good and some are really bad. Now what do I mean by this? My meaning must be a matter of definition and I must define as good that which continues to be accepted through the ages and as bad that which dies rapidly, even in the consciousness of him who invented it.

In the past there have been great men of letters and of music, and these men of music—Bach, Beethoven, Brahms, etc.—had ideas of expression, some doubtless bad, but many good. The good have survived. If there was anything bad, it died long ago. And there have arisen men who have gathered together all these good things and have made out of them rules of composition, rules of harmony, rules of phrasing in music, rules of accents, and many other rules. As a young student, I am presented with the books which these people have written on the subject. At first, my individuality is slightly offended. I exclaim: "I am an artist, and my soul tells me how to play this composition. Why should I be bound by all the exhortations in these musty

old books?" I take a piece of music and play it in a manner which I shall designate as A. My teacher tells me that I should play it in a manner which I will designate as B. I ask why I should play it in the manner B. Contrary to all ideas of modern education in the development of the child's mind, I think the best answer my teacher can give me is frequently: You should play it this way because I tell you to.

And so, if I am obedient, I follow these precepts, at first, possibly, without reason other than that of obedience. Perhaps occasionally I may incorporate one of my own ideas which will pass muster in the sense that it is not frowned upon by what is in the books. By and large, however, I play in obedience to my instructions. Now, if these things which I have been told to do are good things in the sense in which I have defined that term, if they are such things as have been invented by Bach, or Beethoven, it will not be long before my own consciousness absorbs them as part of itself. These ideas are reborn within my own soul, so that again I play as I feel, but this time I play as Bach felt, as Beethoven felt, and as Brahms felt. Indeed, when these things have become thoroughly absorbed in my musical conscience, I am no longer that pig-headed student who started to learn with such confidence. In miniature, I am Bach, I am Beethoven, I am Brahms.

It is of interest to inquire as to the origin of those elements which, in music, and in speech, constitute the good things. I have heard it maintained that the matter goes back to the physiological processes going on in our bodies as a result of the normal activities with which we are associated in our everyday life. Thus, for example, if I play[8] the simple sequence of notes C, D, E, F, G, F, E, D, C, I feel the necessity for an accent on the G, the note on which I reverse my direction of motion in the scale. This reminds me of the fact that if I run forward and then reverse my direction so as to run backwards, I have to give a kind of kick in order to reverse that motion so as to counteract my inertia resulting from the mass of my body. I may say, therefore, that the playing of this element of music calls to mind, probably in subconscious form, the physiological feelings associated with the act I have described. A very common physiological feeling is one of relaxation after effort. Now in music we have come to accept certain simple chords, such as FAC, as things which we can hear without particular emotion. This fact is possibly bound up with a simple relation between the frequencies of the notes concerned, and the corresponding simple relation between the mechanisms and ideas which are stimulated by these notes. On the other hand, if I strike a chord of a more complicated nature, or, to use the technical term, a dissonant chord, such as CEGB♭ the blood seems to go faster through my veins, possibly on account of the greater mechanical and physiological activity going on in my body in the perception of these more complicatedly related frequencies. If again I play this more complicated chord, and follow it by a suitably

[8] These and subsequent similar matters were illustrated on the piano.

chosen simple chord, in this case CFA, I have a feeling of relaxation, possibly because the adjustment in my physiological mechanisms necessary to reach this simple chord after having accommodated themselves to the more complicated chord are of such a nature as to invite repose.

Again, suppose once more I play the simple sequence of notes C, D, E, F, G, F, E, D, C, with its accent at the top. I think you will feel that on my return to the starting point and, in fact, during the notes E, D, C, I have a feeling of repose as in the case of one who has started out on a journey and has come back to the point from which he started. On the other hand, if this person, on arriving home and opening his front door, looking forward to a comfortable siesta, suddenly realizes that his house has been burgled, the normal expectation of repose is frustrated. In an analogous manner, if, after playing my sequence of notes C, D, E, F, G, F, E, D, I am in the expectation of the maximum of repose on return to the note C, I can completely frustrate that expectation by playing on the note C a complicated chord such as CEGB♭.

And so I could go on for many hours calling your attention to possible analogies between emotions which I experience as a result of my everyday actions and those which I can make myself experience through acoustical phenomena.

And so we have for consideration a rather curious postulate to the effect that if a sequence of ordinary and more or less uninteresting physiological experiences is called to our attention, not directly, but through analogy in the utilization of some other form of expression as in the case of sound, or perhaps color, then that form of expression assumes the dignity of an art. It is quite true that if I should call attention in words to the various things which you feel when you run, when you jump, and when you are surprised by some unexpected phenomenon, I should quickly bore you. However, when I call attention to these things in a more subtle manner by analogy in sound, for example, and without your really realizing that I call attention to them, you begin to feel exhilaration in the shadowy recognition of these feelings to which you have become so accustomed in the general activities of life, feelings to whose response you have become more or less blasé and in respect of which you would indeed become irritated, in line with my previous remarks, if your attention were continually called to them in the more obvious form of words. These things, which are the accompaniment of our existence, are fundamentally good things, for indeed it is good to be alive, but in order to appreciate them after prolonged acquaintance with them, they have to be brought to us in a slightly different light, when we shall appreciate them anew, until once more, perchance, we become irritated by a continual recognition of them in this new light.

Mixed up with the natural and good things which play a role in musical expression, there are, as I have implied, bad things which, however, occasionally succeed in becoming sufficiently firmly entrenched

that they live, for a time at any rate, in the imaginations of the unsophisticated. Some of these representatives of false beauty find their birth in the difficulties of the instrument itself. Thus, for example, in stringed instruments one frequently has to pass from one position to another rather remote from it on the same string; and this is most easily accomplished by means of a slide. Now, there may be some places where the character of the music itself demands a slide as a logical part of the expression of things. However, born of a kind of apparent necessity for overcoming the technical difficulty, the slide begins to assume a status of its own which is not limited to its use as part of the logical musical structure. Merely as an acoustical phenomenon, with nothing but one note to start with and another to end with, and in a state of complete dissection from any composition of which the two notes form a part, it comes to acquire an alluring significance, so that unless it is interpolated frequently in anything which is being played, the unfortunate player is accused of having no soul. Even when slides are called for in the music, the line of demarcation between what is beautiful and what, through exaggeration, becomes nauseating, is very narrow. Let me illustrate this matter by playing a fragment with exaggerated slides calculated to send the musician into hysterics but which, if handled properly, can give pleasing effects.[9]

And so, some of these bad things come to have a dramatic significance in their own right and are in the category of the gasp which the operatic tenor is apt to emit on the sudden cessation of his high A. We all know that there are a great many people who will sit through the whole performance in which, if they will be honest with themselves, the one thing they are waiting for is that gasp which they know the tenor will make at the end of the high A. For that they have paid their $2.50 or their $5.00 to hear the whole opera.

But to leave the bad things of musical expression and turning to those elements which are good, it is interesting to note that the potentialities can be characterized under relatively few heads. Much is covered under rhythm, accents, gradations of intensity, intonation, and above all phrasing.

RHYTHM

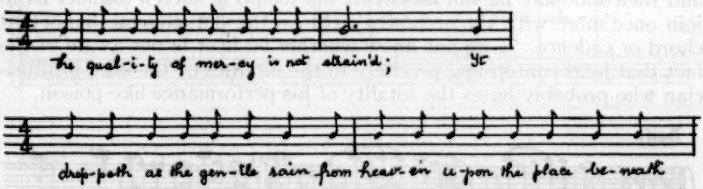
As regards rhythm, the music of our epoch is usually written with time signatures—four four, three four, six eight, and so on. A march is typified by standard four four time, a waltz by three four time, and so forth. Now beauty of rendition is frequently enhanced by slight departures from the rigid metronomic beat. Such departures, characterized by the term rubato, should be made sparingly and according to a well-conceived plan. A good rubato is not one which represents a vague wandering from the strict tempo, but rather a calculated departure from

[9] A recording of the first few bars of the slow movement (Langsam) of the Schuman Concerto for Violoncello and Orchestra was used as an illustration.

that tempo of such a nature that the time which is stolen in one place is made up for in another, so that, over a properly chosen interval characterized by a well-defined phrase or other division in the music, time has not been lost or gained on the whole. Often artifices of this kind, valuable in music, have been consciously or unconsciously recognized in speech where the experience of mankind has had a much longer period in which to develop. Let me take an illustration from the realm of words, as embodied in Portia's speech:

> The quality of mercy is not strain'd;
> It droppeth as the gentle rain from heaven
> Upon the place beneath;

Let us imagine this in perfect regularity, as though it were written to the time four four in music. We should have

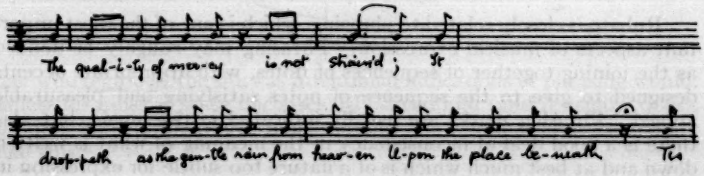


where, as you will see, I have indicated eighth notes to the various syllables in the first bar, for example. If you will beat strict time and recite the line to this tempo, you will naturally find that the result is very primitive and inartistic. On the other hand, if you recite it as you would probably wish to hear it, you could still beat time to it, but the fragment would probably go as follows:[10]

I think if you should examine the various ways in which people would utter this, and subsequent, lines, you would find that while all showed what I may call rubato, those which showed the greatest delicacy of expression would be the ones which conformed to a background of regularity of tempo in the sense in which I have endeavored to explain it.

Now why is it that the utilization of rubato in the form in which I

[10] The fragment was recited in the lecture, but an approximation to the "rubato" is represented by the following:



have presented it results in a pleasing effect? The answer probably goes back to the origins of our physiological structure. Such things have spontaneous appeal even though he to whom they have appeal need not understand the origin of that appeal. Now the radio and television are good places for the testing of fundamental appeal because there it is measured in hard dollars and cents. I do not like crooners, but I must admit that the crooner has appeal for he gets good money and that means appeal. It must therefore be that some things which crooners do have the property of striking right through to those centers of sensitivity in our physiological make-up which are capable of giving us pleasure. Now frequently I have noticed, and I am sure you have, that when a crooner is crooning he will depart in his languid meanderings from the regular tempo, which, however, is being maintained by the orchestra, in such a way that you will feel that he is most assuredly lost, and then suddenly he will accelerate his tempo in such a manner as to join once more with the orchestra right on the dot at some important chord or cadence. I do not know whether he is or is not aware of the fact that he is conforming precisely to the dictates of the kind of musician who probably hates the totality of his performance like poison.

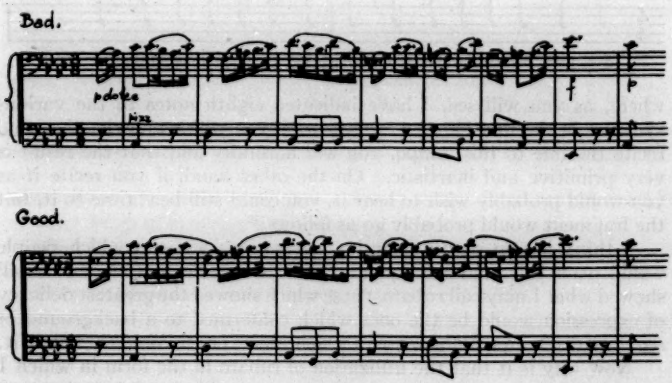



FIG. 3.

PHRASING

Rubato is closely related to phrasing, which is one of the most important aspects of musical expression. Phrasing may roughly be defined as the joining together of sequences of notes, with appropriate accents designed to give to the sequence of notes satisfying and pleasurable appeal. Phrasing marks are usually indicated in the music, but alas there is a good deal of inconsistency in the meaning of what is written down and at best much which is of a nature too subtle for expression in notation remains.

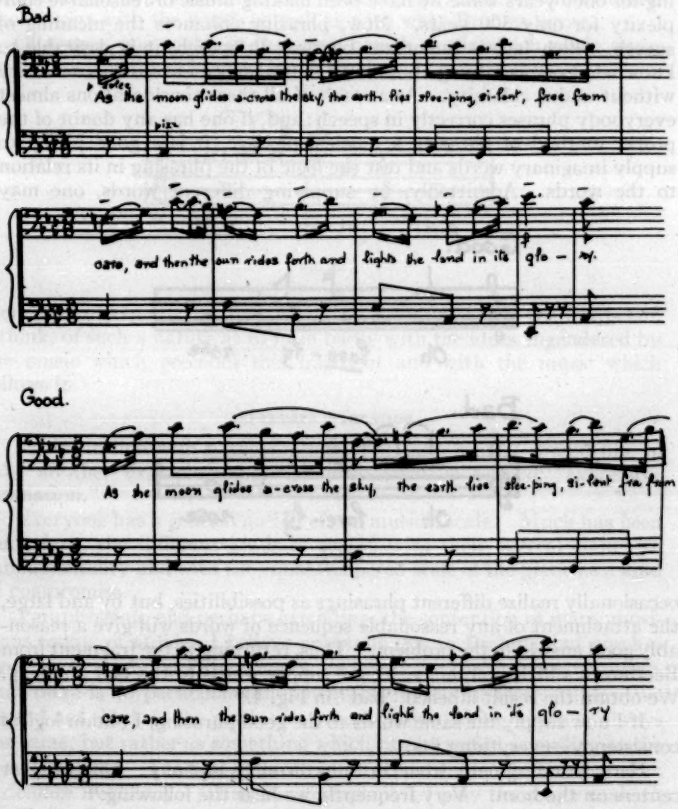
Here (Fig. 3) is a fragment from the *Andante con moto* of the Fifth Symphony of Beethoven. The fragment labelled "bad" is represented as it is usually depicted in the score. I have indicated only the cello and double bass parts which are sufficient to define the situation. Any-

Bad.
As the moon glides a-cross the sky, the earth lies slee-ping, si-lent, free from
care, and then the sun rides forth and lights the land in its glo – ry.
Good.
As the moon glides a-cross the sky, the earth lies slee-ping, si-lent, free from
care, and then the sun rides forth and lights the land in its glo – ry

FIG. 4.

one who regarded the slurs as intentional indications of phrasing would, unless his musical sense unconsciously saved him (and it usually does), produce lamentable results in execution. He would produce something like this.[11]

[11] Here a tape representing the slurs as indicative of phrasing marks was played.

As a contrast, we have what now follows and which would seem to give one form of a proper intent in terms of phrasing as indicated in the fragment labelled "good."

I have already called attention to the fact that we have been speaking for 6000 years while we have been making music of reasonable complexity for only 300 years. Now, phrasing enhances the meaning of speech, which fact is important because, in speech, it is desirable to know what one is talking about. In music one can avoid this necessity without undue calamity. As a result of all these considerations almost everybody phrases correctly in speech; and, if one has any doubt of the proper method of phrasing a musical passage, it is a good policy to supply imaginary words and test the logic of the phrasing in its relation to the words. Admittedly, by supplying different words, one may

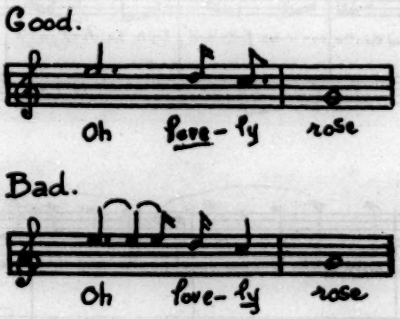


FIG. 5.

occasionally realize different phrasings as possibilities, but by and large, the attachment of any reasonable sequence of words will give a reasonably good answer to the problem. Thus, returning to the fragment from Beethoven's fifth symphony, let me supply words to the bad phrasing. We obtain the result labelled "bad" in Fig. 4.[12]

If I now supply the same words to the good phrasing, I obtain logical consistency—everything fits.

Here is a fragment from Brahms First Symphony. Our interest centers on the horn. Very frequently we hear the following.[13]

Now, in the first place, this is not what Brahms wrote because, in what you have just heard the note C comes on the fourth beat, whereas

[12] In the lecture, the matter was illustrated from a tape. In the illustration, I have attempted to illustrate, in exaggerated form, the bad features, by putting in the kind of accents which the bad phrasing invites.

[13] The illustration on the tape gave the accent to the C which was erroneously sounded on the fourth beat with the one-sixteenth preceding.

Brahms put the D on the fourth beat. If he had intended what you have heard, the D would have preceded the fourth beat.

Attaching words as a test, I hear, in the correct rendering what is represented in the upper form of Fig. 5, and in the bad one I hear what is represented in the lower form.[14] The second rendering is obviously inconsistent with the words as well as with what Brahms wrote.

Of course, one could, in this case, attach words which made sense with what Brahms did not write, in singing

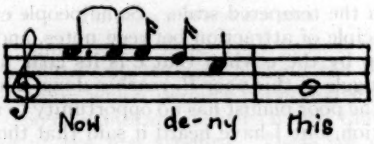


FIG. 6.

Here, apart from the fact that Brahms did not write this, the words are, I think, of such a nature as to fit in badly with the ideas engendered by the music which precedes this fragment and with the music which follows it.

EXPRESSIVE INTONATION

Again, in line with natural elements of expression, I should like to take another example concerned with what is termed "expressive intonation."

Everyone has a general notion of the musical scale. Much has been said about the different kinds of scales, with their several claims to fundamentality and with the equal tempered scale of the piano as a kind of compromise.

Paraphrasing, and perhaps extending, the contention of many musicians today, of an idea I believe originally due to Brahms, but emphasized particularly by Pablo Casals, we assert, for the purposes of music, that there is no particular fundamentality in any of these scales. We should not think of the pitch of a note as a fixed thing which is always the same, but rather as something which can be varied according to the circumstances. The note defines a small acoustical territory rather than a definite frequency. Thus, for example,[15] if I play the major scale C, D, E, F, G, A, B, c, the note B seems anxious to find its home at c. For this reason, it is called the leading note, and this sensitivity shows itself, according to the sensitive musical ear, in a desire to play this B a little sharp when it is followed by the c. On the other hand, if this note

[14] The two renderings were played on a tape.

[15] These matters were illustrated on the piano.

were not followed by c but, while still in the key of C, were part of such a fragment as

FIG. 7.

it would have no such tendency to be sharpened, but would have its normal pitch as in the tempered scale. Some people even go so far as to speak of a principle of attraction between notes, and think of the B as being pulled up by the c when that c is its goal, as though the c, witnessing the struggle of this poor B, reaches down a hand and gives a pull. Of course, the poor pianist has no opportunity to make use of this expressive intonation, but I have heard it said that those pianists who feel very strongly the desire to sharpen the B, for example, as a leading note to c, find themselves satisfying their longing by pressing the B key just a little on the right hand side of the center.

Another aspect of this matter is represented by the resolution of a dissonant, or as I have called it, a complicated chord. Thus, if I play the chord (a), Fig. 8, the rules of its resolution, founded, of course, ulti-

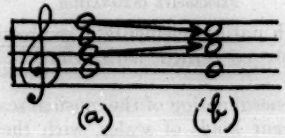


FIG. 8.

mately upon our intuitive feelings, demand that it be followed by the chord (b), in which we may regard the B of the major third as rising to c in the new chord while the seventh of the first chord, namely F, falls to E. Now in the principles of expressive intonation, this F knows, as it were, that it is going to fall to E and is already on its way, so that, in fact, it is played a little flat. On the other hand, the B, as it were, knows that it is going to rise to c. It is already on its way and is played a little sharp. The point is, of course, that the changes in these two cases, that is to say, the fall of the F and the rise of the B, are very important changes in ironing out the dissonance of the first chord into the easy serenity of the second, and we may suppose that it is for this reason that these notes in the first chord are very anxious to be on their way in performing their important duties. Thus we may say that the pitch defines the intent of the future and the memory of the past.

The foregoing matters will serve to illustrate in a general way the

idea of expressive intonation. Now to return to the crooner. On one occasion when, on turning on the radio, I caught one, I listened for a little while and I had to admit that among all the things that I did not like, there was something which sounded good. I set out to find what it was, and lo and behold! I found that the crooner, I believe quite unconsciously, was singing according to the principles of expressive intonation. Perhaps, after all, a tea-party composed of Johannes Brahms, Pablo Casals, and a crooner would not have been such an impossible affair.

THE SEARCH FOR PERFECTION

All the great games which have lived possess those potentialities which permit continued improvement without the attainment of final perfection. Any game which permits perfection loses its charm and becomes a bore. Some games are of such a nature that we can get enjoyment from them at the first attempt. These are usually the superficial games, the games which do not give lasting satisfaction to anyone. They are useful in entertaining us when we are children, but in later years we pass on to better things. The best games inevitably require some period of training, even of drudgery, and the more seriously one builds up the bases for development, the greater the satisfaction he derives. If a richness of potentiality of development is not established in the fundamentals, one is apt to attain before long a kind of pseudo perfection in the sense in which it seems that there is no way in which one can be better, in which one becomes somewhat bored in this condition, and yet disconcerted by a very real paradox in realizing that there are other people who are obviously better at the game for reasons which seem incapable of being defined.

The trouble about the struggle for more and more perfection lies in the fact that we become less and less satisfied with what has pleased us before. The untrained musician who is more or less sensitive to music really gets a very keen enjoyment out of a performance which would give no enjoyment at all to a trained and sensitive musician. What are we to say about this? It is indeed a lamentable fact, which we have to admit, that the uncritically trained person has probably a much wider range of enjoyment than the more critical. It is lamentable, but possibly a fact, that the higher the state of development of a musician the less his potentiality for enjoyment of that which is around to be enjoyed, so that, in the limit, were he sufficiently far advanced, we might suppose that probably he would enjoy nothing, his own performance not excluded. The only compensating feature is that before he reaches this very calamitous stage, and when his enjoyment is limited to a very few experiences, he probably has an intensity of enjoyment during those experiences which outweighs enormously the intensity of enjoyment of the less sophisticated group, whose potentiality for such enjoyment covers so much wider a field.

Sometimes it is a little hard to believe that the highly trained musician can be so disturbed by one or two imperfections. One is apt to feel that he is over-critical, or is endeavoring to convey an air of ultra-superiority. Here again it is well to test ourselves by an appeal to sequences of words, where most of us are better qualified to express an opinion. Suppose I take once again the fragment from Portia's speech in "The Merchant of Venice":

"The quality of mercy is not strain'd;
It droppeth as the gentle rain from heaven
Upon the place beneath;"

Suppose I should replace "droppeth" by "falls down." Would you not be nauseated, and would not the whole speech be spoilt for you in the sense that your nervous system would not have recovered before the speech was over? The change I have cited is rather analogous to something which is the responsibility of the composer in the case of music. He would be the one at fault if something analogous to the use of "falls down" in the place of "droppeth" had been introduced into his composi-

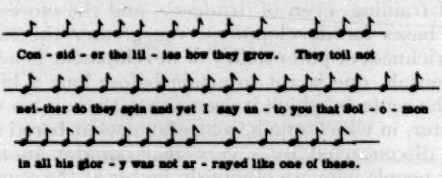

Con - sid - er the lil - ies how they grow. They toil not
nei-ther do they spin and yet I say un - to you that Sol - o - mon
in all his glor - y was not ar - rayed like one of these.

FIG. 9.

tion. To get the feeling of your reaction to the correct *rendering* of the words, however, let us leave the fragment as Shakespeare wrote it, and try making just one major fault in rendering, and see how you feel. Try emphasizing the word "from." I think the beauty of the whole thing will be killed.

In seeking perfection in such an art as music, the goal must be divided into two categories: perfection of concept, and perfection of realization on the particular instrument or instruments on which the music is played. As regards perfection of concept, the beginner will look at the marks written by the composer upon the paper and will perhaps feel that when he has attended to those nothing more can be done. However, very much more can be done.

Now I have no brief for importing into the discussion vague and elusive things like the greatness of the soul of the performer bringing out the unfathomed mysteries of the genius of the composer, and so forth, but I would deal rather with beauties in the form of more or less concrete facts which can be specified to some extent and developed by training,

but which are too subtle and numerous to permit of any nomenclature for their indication upon the music itself. I should like to appeal yet again to the art of speech which has been with us for so many thousands of years. Let us take, slightly condensed, a fragment from the immortal Sermon on the Mount and utter it with perfectly regularly spaced syllables and unchanging intonation—thus (Fig. 9)

This, of course, represents the intolerable, and would correspond in music to somebody playing according to a rhythm indicated, but badly chosen, and without any of the so-called marks of expression. Let us next utter the phrase as a lawyer might be supposed to utter it were he reasoning with somebody in an easy-going manner.[16] Naturally it is difficult to illustrate this matter in writing. As far as we can indicate the intent at all by means of accents (>), intensity gradations ( ), rhythm, and upward and downward inflections of pitch ( ), we might write (Fig. 10), for this case

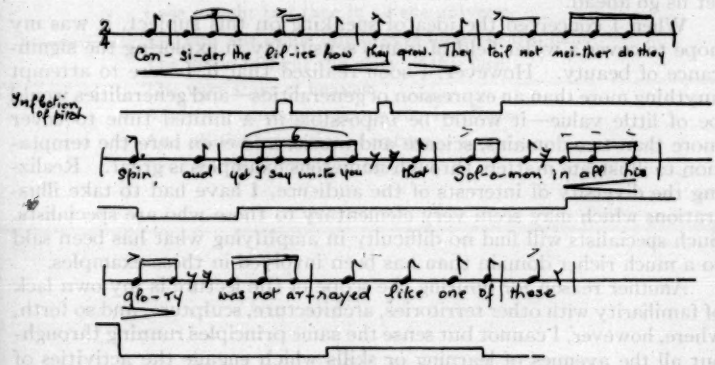

2/4
Con- si-der the lil-ies how they grow. They toil not nei-ther do they
Inflection of pitch
spin, and yet I say un-to you that Sol-o-mon in all his
glo-ry was not ar-rayed like one of those

FIG. 10.

The complexity of the notation so far attained approximates to the requirements of a *thoroughly logical* presentation of the idea. As yet, there cannot be considered to be any beauty in the utterance. No actor would utter and no audience would be content with the utterance of the fragment which I have just cited in the manner in which I have so far spoken it—in the legal form, in fact. Yet suppose I should attempt with greater appeal to speak these words—"Consider the lilies how they grow. They toil not, neither do they spin, and yet I say unto you that Solomon in all his glory was not arrayed like one of these."[17] I think

[16] At this point in the lecture, the lines were uttered in a manner intended to convey the idea, in a manner involving persuasive oratory.

[17] At this point an attempt was made to utter the fragment in such a manner as to impart beauty to the utterance.

it would be very difficult to express in a notation the difference between the last utterance and that which I might call the pure legal utterance. The last utterance is legal also, but it attempts something more.

The teacher can take the student so far in the matter of subtlety of expression and no farther. He can give his pupil certain rules and show him the promised land, but from here on, while he can criticize the performance when made, and can himself illustrate how he would like to see it changed, he can only, by inspiration, hope to bring out from the student something more, if there is something more to be brought out. Uncontrolled, undisciplined expression, however, is as bad or worse than no expression. I am reminded of a case told me by a very dear friend who, on teaching one of his pupils and trying to instruct her as to the manner in which a certain thing should be played, was challenged with the remark: "But, Mr. X, should not an artist play as she feels?" "Sure," said my little friend, "but *you* are not feeling anything. Now let us go ahead."

When I conceived the idea of speaking on this subject, it was my hope to cover a wider field of man's sensitivity in exploring the significance of beauty. However, I soon realized that if I were to attempt anything more than an expression of generalities—and generalities would be of little value—it would be impossible in a limited time to cover more than two domains, science and music, and even here the temptation to illustrate matters through numerous examples is great. Realizing the diversity of interests of the audience, I have had to take illustrations which may seem very elementary to those who are specialists. Such specialists will find no difficulty in amplifying what has been said to a much richer domain than has been involved in these examples.

Another reason for limiting the scope of the lecture is my own lack of familiarity with other territories, architecture, sculpture, and so forth, where, however, I cannot but sense the same principles running throughout all the avenues of learning or skills which engage the activities of man.

Beauty presents herself to us with many facets; but, in the last analysis, the sense of exaltation is very much the same, whatever the medium through which it is conveyed. The ultimate feeling engendered by the solution of a mathematical problem in satisfying form is of the same kind as that engendered in a composer who has brought to clean and successful completion an idea in the realm of his art. No longer in science does one seek theorems bound to the mechanisms of cogs and wheels or to analogies with the waves of the sea or the multifarious activities of machinery. One seeks rather to express nature's laws in forms which are the most beautiful, and that very beauty is symbolic of the logical relationships of things and so reflects a greater power in those laws in their service to us.

And so, in the last analysis, I seem to see this image, this ideal, Beauty, as a goddess who dominates the universe in all its phases. Ye men of letters find the imprint of her majesty in your sense of the power of language, and in the ring of words. Ye men of song find her voice in the harmony of sweet sounds. Ye painters find the richness of her form in your noble designs and in the colors with which you adorn them. Ye men of science find the stamp of her intellect in the great harmonies of nature's mathematical design. And so, if ye should pray for anything, pray that ye may find senses to which all nature's beauties bring response, for then shall ye be as angels. Then shall ye have attained the pinnacle of what may have been the *intent* of the universe, the creation of a realm in which happiness may grow to full maturity.

And if, to this goddess, Beauty, whose breath is the atmosphere which the soul of man again breathes, I should write a poem, I would say:

I see naught but thee in all the universe.
The sun was made but to give luster to thy hair,
The moon to light the glory of thy face.
The very stars of heaven
Are but a canopy to frame thy loveliness
And all the mysteries of nature
Are but the symbols of thy soul.

John Scott Medal Award.—The John Scott Medal Award was established more than a century and a quarter ago by John Scott, an obscure Scottish chemist of Edinburgh, who died in 1816. In his will, he bequeathed the sum of $4000 to the City of Philadelphia, directing that the interest and dividend becoming receivable thereon should "be laid out in premiums to be distributed among ingenious men and women who make useful inventions, but no one of such premiums to exceed twenty dollars and along with which shall be given a copper medal." It is believed that Scott made Philadelphia the instrument of his benefaction because of his admiration for Benjamin Franklin.

From 1821 until 1834, the Philadelphia Society for Promoting Agriculture was in charge of recommending recipients for these premiums. In 1834, The Franklin Institute was authorized by the City of Philadelphia to take over this responsibility. The Institute continued, through its Committee on Science and the Arts, to make such recommendations until 1919, when a separate advisory committee was formed to aid the Board of Directors of City Trusts in selecting candidates for this award.

By Court order in 1920 and 1921, the City of Philadelphia, Trustee, acting by the Board of Directors of City Trusts, was authorized to distribute the income in premiums not exceeding $2000 each. None of the recipients, however, has received more than $1000. From Mr. Scott's original legacy of $4000, the fund has grown to approximately $110,000 to date.

During the 85 years The Franklin Institute was entrusted with the awarding of these premiums, 455 awards were made, including: Thomas A. Edison (mimeograph); Emil Berliner (gramophone); W. S. Burroughs (calculating machine); Thomas Hall (typewriter); C. F. Jenkins (motion picture apparatus); M. and L. E. Levy (screens for photo-mechanical engraving); Sidney Otis *et al.* (electric elevator); J. W. Reno (escalator); E. A. Sperry (gyroscope compass); and Thomas Wood (fountain pen).

Since the Board of City Trusts formed the advisory committee in 1919, other notables have received the John Scott Medal, among them Orville Wright, Madame Curie, Guglielmo Marconi, Lee DeForrest, Irving Langmuir and Sir Alexander Fleming.

In recent years, scientific advancement has resulted more and more through the efforts of research "teams," making it increasingly difficult for the Advisory Committee to recommend individuals who may qualify as recipients of the John Scott Award. The Advisory Committee has, therefore, asked the editors of the JOURNAL to publicize the award, hoping that some of our readers may suggest the names of outstanding individuals who might qualify for the award, which at present consists of $1000 cash, a copper medal and a scroll. Suggestions may be submitted to the editors, who will compile a list of suggested names and forward it to the Advisory Committee.

A NEGLECTED METHOD FOR RESOLUTION OF POLYNOMIAL EQUATIONS

BY

WILLIAM C. TAYLOR, JR.[1]

The problem of finding the irrational zeros of polynomials is of great importance in applied mathematics. It has led to the use of analog computers and nomographs to avoid computational drudgery. Several methods are customarily presented in any first course on the theory of equations, but not all yield the solution with equal facility. The method described here seems to possess merit because (1) it is applicable to a wide variety of problems, and (2) it is greatly facilitated by the use of modern desk calculators.

THE METHOD OF DANIEL BERNOULLI

$$x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = 0 \quad (a_i \text{ real}) \tag{1}$$

$$y_{t+n} + a_{n-1}y_{t+n-1} + \cdots + a_0y_t = 0. \tag{2}$$

The solution of (2) is known to be (1, p. 165):[2]

$$y_t = c_1x_1^t + c_2x_2^t + \cdots + c_nx_n^t, \tag{3}$$

where x_i $(i = 1, 2, \cdots, n)$ are the roots of (1) and c_i $(i = 1, 2, \cdots, n)$ are arbitrary functions of t of period 1 (2, pp. 178 ff).

If $|x_1|$ is greater than the modulus of any other root, the first term in (3) becomes very large compared with the other terms when t is large, and therefore we have

$$x_1 = \lim_{t\to\infty} \frac{y_{t+1}}{y_t}. \tag{4}$$

Hence, Bernoulli's rule:[3]

In order to find the absolutely greatest root of Eq. 1, we take any arbitrary values for $y_0, y_1, \cdots, y_{n-1}$. From these by repeated application of (2) we calculate in succession the values of $y_n, y_{n+1}, y_{n+2}, \cdots$. The ratio of two successive members of this sequence will tend to a limit, which limit is the absolutely greatest root of Eq. 1, provided such root is real and there do not exist roots of equal magnitude but opposite in sign. If this ratio oscillates and the limit (4) does not exist, $|x_1|$, = max $|x_i|$, is non-real or $x_1 = -x_2$.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] Daniel Bernoulli, *Commentarii Acad. Sc. Imp. Petropol.*, Vol. III, p. 1732 (1728).

It is readily proved that (4) will hold even for multiple roots. In the event that $x_1 = -x_2$, the ratio of (4) becomes

$$\frac{x_1 + \frac{c_2}{c_1} x_1(-1)^{t+1} + \cdots}{1 + \frac{c_2}{c_1}(-1)^t + \cdots}$$

and no limit is reached. However, writing y_{t+2}/y_t we get a converging sequence, thus

$$\lim_{t \to \infty} y_{t+2}/y_t = x_1^2. \tag{5}$$

This formula holds equally well if x_1, x_2 are conjugate pure imaginaries, this being evidently the case when the limit of (5) is less than zero.

To determine the absolutely smallest root, x_n, we need only make the transformation

$$x = X^{-1}$$

in Eq. 1, thus

$$a_0 X^n + a_1 X^{n-1} + \cdots + 1 = 0. \tag{6}$$

The method is then applied as before, x_n, the desired root, being the reciprocal of the X_1 for which we solve.

Usually the process of determining x_1 is begun by taking

$$y_0 = y_1 = \cdots = y_{n-2} = 0$$

and

$$y_{n-1} = 1.$$

The difference equation (2) will then give

$$y_n = -a_{n-1};$$

$$y_{n+1} = -a_{n-1}y_n - a_{n-2};$$

$$y_{n+2} = -a_{n-1}y_{n+1} - a_{n-2}y_n - a_{n-3};$$

and so on. A variation of assigned values of y_i ($i = 1, 2, \cdots, n-1$), to speed convergence, where required, will be demonstrated in the following examples.

Illustrative Examples

$$f(x) = x^3 - 5x^2 - 2x + 1 = 0$$

$$y_0 = y_1 = 0;\quad y_2 = 1;\quad -a_{n-1} = 5;\quad -a_{n-2} = 2;\quad -a_{n-3} = -1$$

$t =$	0	1	2	3	4	5	6	7	8	.....
	5			5	25	135	720	3,845	20,530	.....
		2		0	2	10	54	288	1,538	.....
			−1	0	0	−1	−5	−27	−144	.....
$y_t =$	0	0	1	5	27	144	769	4,106	21,924	.....

$$x_1 = y_8/y_7 = 5.339$$

In the foregoing example we note that

$$f(5)\cdot f(6) < 0, \quad \text{hence} \quad x_1 : (5, 6).$$

We may then assign arbitrarily: $y_0 = 1$; $y_1 = 5$; $y_2 = 5^2$. The total number of computations for the same precision is lessened, thus:

125	670	3,575	$x_1 = 3818/715 = 5.339$
10	50	268	
−1	−5	−25	...
134	715	3,818	...

Real root of least modulus

Given

$$x^3 = 2x^2 - 5x + 1.$$

2	4	− 2	−22	−30	
	−5	−10	5	55	
		1	2	− 1	
2	−1	−11	−15	24	...

The non-monotone nature of the sequence y_t indicates that a limit is not likely to be reached for the quotient y_{t+1}/y_t. Make the transformation $x = 1/p$, and solve for x_3:

$$p^3 = 5p^2 - 2p + 1$$

5	25	115	
	−2	−10	
		1	
5	23	106	...

$$x_3 = 1/p = 23/106 = 0.217.$$

Since the assignment of the first $n-1$ differences is arbitrary, the use of approximate numbers does not present difficulties. The following elementary problem illustrates the use of the slide rule with Bernoulli's method:

Problem.—Determine the currents (Fig. 1) when the switch is closed,

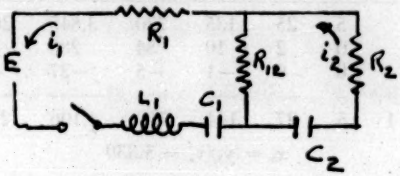

FIG. 1.

given
$$R_1 = 10 \text{ ohms}, \quad R_{12} = 5 \text{ ohms},$$
$$R_2 = 3 \text{ ohms}, \quad L_1 = 0.01 \text{ henry},$$
$$C_1 = 0.000001 \text{ farad},$$
$$C_2 = 0.0000005 \text{ farad}.$$

$$R_1 i_1 + R_{12} i_1 - R_{12} i_2 + (1/C_1) \int i_1\, dt + L_1\, di_1/dt = E$$

$$R_2 i_2 + (1/C_2) \int i_2\, dt + R_{12} i_2 - R_{12} i_1 = 0.$$

Differentiating with respect to time,

$$(R_1 + R_{12})\, Di_1 + (1/C_1) i_1 + L_1 D^2 i_1 - R_{12} i_2 = 0$$
$$- R_{12}\, Di_1 + [(R_2 + R_{12})\, D + 1/C_2] i_2 = 0.$$

Then,

$$\begin{vmatrix} L_1 D^2 + (R_1 + R_{12})\, D + 1/C_1 & - R_{12} D \\ - R_{12} D & (R_2 + R_{12})\, D + 1/C_2 \end{vmatrix} = 0.$$

Expanding the determinant and substituting the given values, we have:

$$8 \times 10^{-2} D^3 + 20{,}095\, D^2 + 38 \times 10^6\, D + 2 \times 10^{12} = 0,$$

or

$$- D^3 = 251 \times 10^3\, D^2 + 475 \times 10^6\, D + 25 \times 10^{12}$$

251×10^3	63×10^9	156.9×10^{14}	39.2×10^{20}
..	$-0.475(10^9)$	-1.19×10^{14}	-0.297×10^{20}
..	..	0.25×10^{14}	6.28×10^{20}
251×10^3	62.5×10^9	155.96×10^{14}	39.183×10^{20}

$$D = -250(10^9) \qquad = -251(10^9).$$

The value -250×10^3 is closer than -251×10^3. Dividing, we get $D^2 + 10^3 D + 225 \times 10^6 = 0$ (approx.). This gives

$$D = -500 \pm j\,30{,}000, \quad \text{etc.}$$

COMPARISON WITH OTHER METHODS

Newton's Equation is used in many textbooks for illustration of methods of resolution. The solution (for greatest root) by Bernoulli's method, is as follows:

Given: $x^3 - 2x - 5 = 0$. $f(2) \cdot f(3) < 0$ $\therefore\ x_1 : (2, 3)$

First step:

$x = 2 + p$

$q = 1/p$

$q^3 = 10q^2 + 6q + 1$

			1	0	−2	−5	2
			1	2	2	−1	
			1	4	10		
			1	6			
10	100	1,060	11,210	118,560	1,253,920	.....	
	6	60	636	6,726	71,136	.....	
		1	10	106	1,121	.....	
10	106	1,121	11,856	125,392	1,326,177	.....	

$$p = \frac{125{,}392}{1{,}326{,}177} = 0.0945514814.$$

Hence

$$x_1 = 2.094551481$$

correct to nine decimal places.

For comparison see, for example: Dickson's solution by Horner's method (6, pp. 87–89); the method of iteration illustrated by Uspensky (7, p. 171); the Newton-Raphson method (1, pp. 489–492); Steffenson's "Inverse Interpolation" (2, pp. 84–85), and Jordan's method of false position (1, pp. 488–489). Particularly if the columns exhibited above are calculated with the aid of the cumulative multiplier key on an electric desk calculator, one cannot fail to be impressed with the ease of the Bernoulli solution.

It is not the contention of the writer that this method can supplant the others, but it is felt that this neglected method ought to be taught inasmuch as there are many problems for which it is the quickest and easiest process. In particular, since the cubic equation will always have at least one real root (either greater or less modulus), the Bernoulli solution (with the aid of a desk calculator) will always be applicable.

REFERENCES

(1) CHARLES JORDAN, *Calculus of Finite Differences*, 2d Ed., New York, Chelsea Publishing Company, 1950.

(2) J. F. STEFFENSON, *Interpolation*, Baltimore, Chelsea Publishing Company, 1929.

*(3) E. T. WHITTAKER and G. ROBINSON, *The Calculus of Observations*, 2d Ed., London and Glasgow, Blackie & Son, Ltd., 1937.

*(4) A. C. AITKEN, papers in the "Proceedings of the Royal Society of Edinburgh": V. 46, pp. 289–305 (1926); V. 51, pp. 80–90 (1931); V. 57, pp. 269–304 (1936).

*(5) J. B. SCARBOROUGH, *Numerical Mathematical Analysis*, Baltimore, The Johns Hopkins Press, 1950.

(6) L. G. DICKSON, *First Course in Theory of Equations*, New York, John Wiley and Sons, 1922.

(7) J. V. USPENSKY, *Theory of Equations*, New York, McGraw-Hill Book Company, 1948.

* Not referred to in this paper, but suggested for further reading. Reference 4 was given to the writer by Prof. W. H. Spragens and is quoted in Householder, Alston S., "Principles of Numerical Analysis," New York, McGraw-Hill Book Co., 1953.

SYMMETRICALLY LOADED CIRCULAR PLATES *

BY

M. STIPPES [1] AND R. E. BECKETT [2]

INTRODUCTION

The linear solution for plate problems yields deflections in excess of the true deflection for a given load (maximum values are implied here) and the amount of this departure increases with the load on the plate. In 1910, von Karman presented a set of "small-large" deflection equations in which the first order effects of the membrane stresses were taken into account. At that time, he also indicated the difficulty involved in solving his set of equations which are non-linear in character. A number of authors have presented, in the literature, a variety of procedures for obtaining solutions to the various boundary value problems associated with the above mentioned equations. In particular, Stewart Way, in 1937, exhibited a method for solving the problem of a symmetrically loaded circular plate by expanding the stresses and deflections in a power series in the radius. His approach, of course, requires that the loading on the plate be an analytic function of the radius.

We present here a procedure which only requires that the slope in the corresponding linear problem be expandable in either a Fourier-Bessel series or a Dini series depending on the nature of the boundary conditions involved. This attack is considerably more general than Way's method since the type of functions expandable in terms of the above mentioned series includes most if not all of the loads of practical interest. Additionally, it is felt by the authors that, starting with the results of this paper, the computational efforts are considerably reduced. Of interest also is the fact that certain properties of the stresses in the plate can be obtained without a knowledge of the exact solution of the problem. These ideas are discussed at some length in the body of the paper.

OUTLINE OF THE METHOD

We begin by considering von Karman's equations for radially symmetric deflections of a circular plate. In self-adjoint form and reduced

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dimensionally, these equations are[3]

$$\rho \nabla_\rho^4 \phi = -\frac{1}{2}\frac{d}{d\rho}\left(\frac{d\psi}{d\rho}\right)^2$$
$$\rho \nabla_\rho^4 \psi = \lambda \rho f(\rho) + 12(1-\sigma^2)\frac{d}{d\rho}\left(\frac{d\phi}{d\rho}\frac{d\psi}{d\rho}\right). \tag{1}$$

(∇_ρ^4 is the usual iterated Laplacian in two dimensions in which all dependence on θ has been suppressed and a is the radius of the plate.) To obtain definite solutions to Eqs. 1 we demand that the dependent variables ϕ and ψ satisfy certain conditions at $\rho = 0$ and $\rho = 1$. For the present endeavor, we shall restrict ourselves to the classical boundary conditions. We require then that ϕ and ψ be regular at the origin. At $\rho = 1$, one of the following four sets of conditions may hold true:

$$\begin{array}{ll} \text{I.} \quad a \;\&\; c & \text{II.} \quad a \;\&\; d \\ \text{III.} \quad b \;\&\; c & \text{IV.} \quad b \;\&\; d \end{array} \tag{2}$$

where

(a) is $\phi = 0$, $\dfrac{1}{\rho}\dfrac{d\phi}{d\rho} = 0$; zero radial (membrane) stress

(b) is $\phi = 0$, $\rho^{-1}\left\{\rho\dfrac{d^2\phi}{d\rho^2} - \sigma\dfrac{d\phi}{d\rho}\right\} = 0$; zero radial displacement

(c) is $\psi = 0$, $\dfrac{d\psi}{d\rho} = 0$; clamped

(d) is $\psi = 0$, $\rho^{-1}\left\{\rho\dfrac{d^2\psi}{d\rho^2} + \sigma\dfrac{d\psi}{d\rho}\right\} = 0$; simply-supported.

The two distinct types of conditions (a) and (b) for either of the second two arise from the manner in which the second two are achieved. For example, a clamped edge can be fixed in a horizontal slider which offers no resistance to motion in the plane of the plate; consequently, boundary conditions (a) prevail. On the other hand, if the piston is prevented from moving in the horizontal direction, (b) would prevail. It may be observed at this point that the choice of ϕ as zero on the boundary is in no way a limitation since ϕ enters into the differential equations and physical boundary conditions only through its derivatives. We also remark that because of symmetry and the nature of plate problems both $d\phi/d\rho$ and $d\psi/d\rho$ vanish at $\rho = 0$.

The method of solution will require the Green's function for

$$\rho \nabla_\rho^4 \Delta = 0 \tag{3}$$

[3] This form may be obtained from Eqs. 200 and 201, p. 343 of Timoshenko's "Theory of Plates and Shells," New York, McGraw-Hill Book Co., 1940, by letting

$$x^2 + y^2 = a^2\rho^2, \quad F = Eh^3\phi, \quad w = h\psi, \quad \lambda = qa^4/Dh.$$

subjected to one or the other of (*a*), (*b*), (*c*), or (*d*), hence we catalog these functions at this point:

$$\Phi^a(\rho, \xi) = \frac{(1 - \xi^2)(1 + \rho^2)}{8} + \frac{(\rho^2 + \xi^2)\log \xi}{4} \qquad \rho < \xi$$

$$\Phi^b(\rho, \xi, \sigma) = \frac{(1 - \xi^2)[(3 - \sigma) - (1 + \sigma)\rho^2]}{8(1 - \sigma)} + \frac{(\rho^2 + \xi^2)\log \xi}{4} \qquad \rho < \xi \tag{4}$$

$$\Psi^c(\rho, \xi) \equiv \Phi^a(\rho, \xi)$$

$$\Psi^d(\rho, \xi) \equiv \Phi^b(\rho, \xi, -\sigma).$$

Since $\rho \nabla_\rho^4$ is a self-adjoint operator, all of the Green's functions of (3) are symmetric functions of their variables. Hence one obtains the values of (4) for $\rho > \xi$ merely by interchanging ρ and ξ in (4). In addition, the second derivatives of (4) with respect to ρ and ξ will be needed. Again because of symmetry we need only

$$\begin{aligned}
\frac{\partial^2 \Phi^a}{\partial \rho\, \partial \xi} &= K^a(\rho, \xi) = \frac{\rho}{2\xi}(1 - \xi^2) \qquad \rho < \xi \\
\frac{\partial^2 \Phi^b}{\partial \rho\, \partial \xi} &= K^b(\rho, \xi, \sigma) = \frac{\rho}{2\xi}(1 + \nu\xi^2) \qquad \rho < \xi \quad \nu = \frac{1 + \sigma}{1 - \sigma} \\
\frac{\partial^2 \Psi^c}{\partial \rho\, \partial \xi} &= K^a(\rho, \xi) \\
\frac{\partial^2 \Psi^d}{\partial \rho\, \partial \xi} &= K^b(\rho, \xi, -\sigma),
\end{aligned} \tag{5}$$

By utilizing the properties of the Green's function, the system (1) and any of the appropriate boundary conditions can be reduced to

$$\psi(\rho) = \lambda \int_0^1 \xi f(\xi) \Psi(\rho, \xi)\, d\xi + 12(1 - \sigma^2) \int_0^1 \Psi(\rho, \xi) \frac{d}{d\xi}\left(\frac{d\phi}{d\xi}\frac{d\psi}{d\xi}\right) d\xi. \tag{6}$$

A differentiation with respect to ρ and a few integrations by parts produce the following non-linear integral equation for the slope

$$\frac{d\psi}{d\rho} = \lambda \int_0^1 \xi f(\xi) \frac{\partial \Psi}{\partial \rho}\, d\xi - 6(1 - \sigma^2) \int_0^1 \int_0^1 L(\rho, \xi) K(\xi, \bar{\xi}) \left(\frac{d\psi}{d\bar{\xi}}\right)^2 \frac{d\psi}{d\xi}\, d\xi\, d\bar{\xi} \tag{7}$$

where Ψ, K, and L are suitably chosen from (4) and (5) depending on the choice of (2).

Now if $f(\xi)$ is such that the first integral on the right hand side of (7) is such that

$$\int_0^1 \xi f(\xi) \frac{\partial \Psi}{\partial \rho}\, d\xi = F(\rho) \tag{8}$$

is of bounded variation and further if

$$\int_0^1 \sqrt{\rho}\,|F(\rho)|\,d\rho \tag{9}$$

exists, then $d\psi/d\rho$ is expandable in a series of Bessel functions of order one. The reason for choosing Bessel functions of order one will become apparent when we discuss the nature of (7). This Bessel series will either be of the Fourier-Bessel type if condition (c) holds on ψ or it will be a Dini type series if condition (d) holds on ψ. It may be noted at this point that the function $F(\rho)$ represents the slope for the linear problem.

Now in view of the nature of the operator $\rho\,\nabla_\rho^4$, the functions K, L are Green's functions for certain second order equations and hence will have expansions in terms of the eigen-functions of these equations (Mercer's Theorem). Indeed, we have

$$K^a(0,\xi) = K^a(1,\xi) = 0 \qquad J_1(\alpha_n) = 0$$

$$K^a(\rho,\xi) = \sum_{n=1}^{\infty} A_n J_1(\alpha_n\rho) J_1(\alpha_n\xi) \qquad A_n = \left[\frac{1}{\sqrt{2}}\,\alpha_n J_0(\alpha_n)\right]^{-2}$$

$$K^b(0,\xi) = 0 \qquad \frac{1}{\rho}\left\{\rho\frac{dK^b}{d\rho} - \sigma K^b\right\}_{\rho=1} = 0 \tag{10}$$

$$K^b(\rho,\xi) = \sum_{n=1}^{\infty} B_n J_1(\beta_n\rho) J_1(\beta_n\xi) \qquad B_n = \frac{1}{\dfrac{\beta_n^2}{2}\left[J_1^2(\beta_n) - J_0(\beta_n)J_2(\beta_n)\right]}$$

$$\frac{1}{\beta_n}\left[\beta_n J_1'(\beta_n) - \sigma J_1(\beta_n)\right] = 0$$

$$L^c(\rho,\xi) = K^a(\rho,\xi)$$

$$L^d(0,\xi) = 0 \qquad \frac{1}{\rho}\left[\rho\frac{dL^d}{d\rho} + \sigma L^d\right]_{\rho=1} = 0$$

$$L^d(\rho,\xi) = \sum_{n=1}^{\infty} C_n J_1(\gamma_n\rho) J_1(\gamma_n\xi) \qquad C_n = B_n(\gamma_n)$$

$$\gamma_n^{-1}\{\gamma_n J_1'(\gamma_n) + \sigma J_1(\gamma_n)\} = 0.$$

With these expansions we can now rewrite (7) as

$$\frac{d\psi}{d\rho} = \lambda \sum_{n=1}^{\infty} F_n J_1(\mu_n\rho) - 6(1-\sigma^2) \sum_{n=1}^{\infty} D_n J_1(\mu_n\rho) \times \sum_{m=1}^{\infty} E_m \left(\int_0^1 J_1(\mu_n\xi) J_1(\lambda_m\xi) \frac{d\psi}{d\xi}\, d\xi \right) \left(\int_0^1 J_1(\lambda_m\xi) \left(\frac{d\psi}{d\xi} \right)^2 d\xi \right). \quad (11)$$

Since the coefficients of $J_1(\mu_n\rho)$ are pure numbers, this now suggests the possibility of assuming the following form for $d\psi/d\rho$ as

$$\frac{d\psi}{d\rho} = \sum_{n=1}^{\infty} \psi_n J_1(\mu_n\rho). \quad (12)$$

A substitution of this into (7) and an appeal to the identity theorem for Bessel series yield the following infinite set of cubic equations for the determination of the ψ_n:

$$\psi_n = \lambda F_n - 6(1-\sigma^2) D_n \sum_{m=1}^{\infty} E_m \int_0^1 J_1(\mu_n\xi) J_1(\lambda_m\xi) \times \left(\sum_{k=1}^{\infty} \psi_k J_1(\mu_k\xi) \right) d\xi \int_0^1 J_1(\lambda_m\bar{\xi}) \left(\sum_{p=1}^{\infty} \psi_p J_1(\mu_p\bar{\xi}) \right)^2 d\bar{\xi}. \quad (13)$$

It is to be noticed here that the complete specification of these equations demands an evaluation of integrals of the following type:

$$\int_0^1 \xi f(\xi) J_1(\mu_n\xi)\, d\xi$$
$$S(a,b,c) = \int_0^1 J_1(\mu_a\xi) J_1(\mu_b\xi) J_1(\mu_c\xi)\, d\xi. \quad (14)$$

The first set of integrals depends on the nature of the loading and must be computed in each specific case. On the other hand, the second set of integrals depends only on the nature of the boundary conditions. Hence once these are tabulated, they remain good for all types of loading.

With the ψ_n so determined from (13) we have the slope at any point in the plate. Furthermore, since $\psi = 0$ for any of the boundary conditions considered at $\rho = 1$, a well-known theorem permits us to integrate (12) term by term. The deflection is then given by

$$\psi(\rho) = -\sum_{n=1}^{\infty} \frac{\psi_n}{\alpha_n} [J_0(\alpha_n\rho) - J_0(\alpha_n)]. \quad (15)$$

THE ROOTS OF $\rho^{-1}[\rho J_1'(\rho) \pm \sigma J_1(\rho)] = 0$

From the recurrence formulae of the Bessel functions we have

$$J_0(\rho) = \frac{1}{\rho} J_1(\rho) + J_1'(\rho) \quad (16)$$

therefore we consider instead the equations

$$\rho^{-1}\{\rho J_0 - (1 \mp \sigma)J_1\} = 0. \tag{17}$$

If we choose $\sigma = 1$, then we have the equations

$$\begin{aligned} 3\rho J_0 - 2J_1 &= 0 \\ 3\rho J_0 - 4J_1 &= 0. \end{aligned} \tag{18}$$

The first ten positive roots of (18a) were tabulated by Airey who also gave the following asymptotic expansion for the roots

$$\rho = \beta - \frac{13}{3(8\beta)} - \frac{2604.89}{9(8\beta)^3} - \frac{445{,}908}{27(8\beta)^5} + 0\left(\frac{1}{\beta^7}\right) \tag{19}$$

where

$$\beta = (4n + 3)\frac{\pi}{4}. \tag{20}$$

Table I gives Airey's values for the first ten positive roots of (18a):

TABLE I.—*Roots of* $[3\rho J_0(\rho) - 2J_1(\rho)] = 0$.

γ_1	2.0694	γ_6	18.0241
γ_2	5.3958	γ_7	21.1802
γ_3	8.5758	γ_8	24.3251
γ_4	11.7346	γ_9	27.4692
γ_5	14.8861	γ_{10}	30.6128

For the computation of the roots of (18b), one can again give an asymptotic formula for the roots as

$$\rho = \beta - \frac{29}{3(8\beta)} - \frac{74{,}260}{81}\frac{1}{(8\beta)^3} - \frac{490{,}160{,}352}{3{,}645}\frac{1}{(8\beta)^5} - 31{,}311{,}905.51\frac{1}{(8\beta)^7} + 0\left(\frac{1}{\beta^9}\right) \tag{21}$$

where

$$\beta = (4n - 1)\frac{\pi}{4} \tag{22}$$

which produces the first ten roots (Table II).

INTEGRATION OF $S(a, b, c)$

The tabulation of the function $S(a, b, c)$ is the most laborious of the computational tasks involved in this method. Some consolation lies in the fact that these functions do not have to be recomputed for each

load considered but only for the type of boundary conditions assumed. Hence once a set of tables of $S(a, b, c)$ is presented for the four possible combinations of boundary conditions, a tremendous amount of work toward solving any problem will have been completed.

TABLE II.—*Roots of* $[3\rho J_0(\rho) - 4J_1(\rho)] = 0$.

β_1	1.545022	β_6	17.996960
β_2	5.266303	β_7	21.148580
β_3	8.496650	β_8	24.297589
β_4	11.677292	β_9	27.444892
β_5	14.841047	β_{10}	30.591017

(Note: the first two roots were obtained from the Harvard Tables by interpolation.)

We begin by noticing that the value of the function $S(a, b, c)$ does not depend on the order in which its arguments a, b, c occur but only on the combination of the three values. This, of course, reduces the size of the tables involved. For the purpose of rapidity of convergence we presume that the arguments a, b, c are ordered in the following manner $a \leqslant b \leqslant c$. If we introduce the notation

$$A_n = J_n(ax) \quad B_n = J_n(bx) \quad C_n = J_n(cx) \tag{23}$$

then we have

$$S(a, b, c) = \int_0^1 A_1 B_1 C_1 \, dx \tag{24}$$

which may be rewritten as follows:

$$S(a, b, c) = \int_0^1 \frac{A_1}{x} \frac{B_1}{x} x^2 C_1 \, dx. \tag{25}$$

An integration by parts now yields

$$\begin{aligned} S(a, b, c) &= \bar{A}_1 \bar{B}_1 \bar{C}_2 + \frac{1}{c} \int_0^1 \left(a \frac{A_2}{x} \frac{B_1}{x} + b \frac{A_1}{x} \frac{B_2}{x} \right) x^2 C_2 \, dx \\ &= \bar{A}_1 \bar{B}_1 \bar{C}_2 + \lambda \int_0^1 \frac{A_2}{x^2} \frac{B_1}{x} x^3 C_2 \, dx + \mu \int_0^1 \frac{A_1}{x} \frac{B_2}{x^2} x^3 C_2 \, dx \end{aligned} \tag{26}$$

where we have introduced the additional notation

$$\bar{A}_n = J_n(a); \quad \bar{B}_n = J_n(b); \quad \bar{C}_n = \frac{J_n(c)}{c}; \quad \lambda = \frac{a}{c}; \quad \mu = \frac{b}{c}. \tag{27}$$

Repeated application of integration by parts produces the following

series expansion for the integral

$$S(a, b, c) = \bar{A}_1\bar{B}_1\bar{C}_2 + (\lambda\bar{A}_2\bar{B}_1 + \mu\bar{A}_1\bar{B}_2)\bar{C}_3 + (\lambda^2\bar{A}_3\bar{B}_1 + 2\lambda\mu\bar{A}_2\bar{B}_2 + \mu^2\bar{A}_1\bar{B}_3)\bar{C}_4 + \cdots + \left\{\sum_{k=0}^{n}\binom{n}{n-k}\lambda^{n-k}\mu^k\bar{A}_{n-k+1}\bar{B}_{k+1}\right\}\bar{C}_{n+2} + \cdots. \quad (28)$$

We now show that this series is convergent (absolutely) for all values of a, b, c. For indeed, we have

$$|S(a, b, c)| \leqslant |\bar{A}_1\bar{B}_1|\bar{C}_2 + \cdots + \left\{\sum_{k=0}^{n}\binom{n}{n-k}\lambda^{n-k}\mu^k|\bar{A}_{n-k+1}\bar{B}_{k+1}|\right\}|\bar{C}_{n+2}| \leqslant \sum_{n=2}^{\infty}(\lambda+\mu)^{n-2}|\bar{C}_n|. \quad (29)$$

But now for fixed c, $\theta_n \to 0$ as $n \to \infty$ in the expression

$$J_n(c) = \frac{(\frac{1}{2}c)^n}{n!}(1 + \theta_n). \quad (30)$$

If we assume N is such that $|\theta_n| < 1$ for all $n > N$ we may write

$$|S(a, b, c)| \leqslant \sum_{n=2}^{N}(\lambda+\mu)^{n-2}|\bar{C}_n| + \sum_{n=N+1}^{\infty}(\lambda+\mu)^{n-2}\frac{(\frac{1}{2}c)^n}{n!}(1+\theta_n)\frac{1}{c} \leqslant \sum_{n=2}^{N}(\lambda+\mu)^{n-2}|\bar{C}_n| + \sum_{n=N+1}^{\infty}\frac{(\lambda+\mu)^{n-2}\left(\frac{c}{2}\right)^{n-1}}{n!}. \quad (31)$$

Now the remainder can be written in the form (assuming $a \leqslant b \leqslant c$)

$$\sum_{n=N+1}^{\infty}\frac{(\lambda+\mu)^{n-2}}{n!}\left(\frac{c}{2}\right)^{n-1} \leqslant \frac{1}{c}\sum_{n=N+1}^{\infty}\frac{c^n}{n!} \leqslant \frac{c^N e^c}{(N+1)!}. \quad (32)$$

Hence we have that

$$\lim_{N\to\infty}|S(a, b, c) - S_N(a, b, c)| = \lim_{N\to\infty}\frac{c^N e^c}{(N+1)!} = 0. \quad (33)$$

Consequently, the series expression (28) converges for all a, b, c. It would be desirable, from the point of view of a computor, to have a reasonably sharp estimation of the error involved in truncating (28) after n terms. This, the authors were unable to attain. Additionally,

the convergence of the series is slow when two of the values of the arguments are equal and greater than the third.

Examples

To illustrate the method we present two examples, which can be solved by Way's approach but present tremendous algebraic complexities.

The first of these examples is the problem of a plate uniformly loaded over a circle of radius b and zero load for $r > b$ with the set of boundary conditions III. In this case we have

$$\begin{aligned} F(\rho) &= \frac{\rho^3}{16} + \frac{\rho}{4}\left[\mu^2 \log \mu - \frac{\mu^4}{4}\right]; \quad \rho < \mu \quad \mu = b/a \\ F(\rho) &= \frac{\mu^4}{16}\frac{(1-\rho^2)}{\rho} + \frac{\mu^2}{4}\rho \log \rho; \quad \rho > \mu. \end{aligned} \tag{34}$$

Accordingly, we write

$$F(\rho) = \sum_{n=1}^{\infty} F_n J_1(\alpha_n \rho). \tag{35}$$

Computing the coefficients F_n by the usual orthogonality properties of the Bessel functions, we obtain

$$F_n = \frac{J_1(\alpha_n\mu)\left[\frac{\mu^4}{8\alpha_n^2} - \frac{\mu^3}{8\alpha_n^2} - \frac{\mu^2}{\alpha_n^4}\right] + J_0(\alpha_n\mu)\left[\frac{\mu^3}{2\alpha_n^3} - \frac{\mu^2}{2\alpha_n^3}\right] + \frac{\mu^2}{2\alpha_n^3} J_0(\alpha_n)}{\int_0^1 \rho J_1^2(\alpha_n\rho)\, d\rho}. \tag{36}$$

If now we take

$$\frac{d\psi}{d\rho} = \sum \psi_n J_1(\alpha_n \rho) \tag{37}$$

and substitute this into (7), the same set of equations is obtained as given in Eq. 13. The solution of this set of equations is most easily effected by iteration.

In the second example, we consider the case of a concentrated load P acting at the center of the plate. Hence, we have

$$\lambda F(\rho) = \frac{Pa^2}{4\pi Dh}\rho \log \rho = \bar{\lambda}\rho \log \rho \tag{38}$$

and

$$F_n = \frac{2}{\alpha_n^3}[J_0(\alpha_n) - 1] \Big/ \int_0^1 \rho J_1^2(\alpha_n\rho)\, d\rho. \tag{39}$$

APPRAISALS FOR STRESSES AND DEFLECTIONS

We begin by assuming that Eq. 7 has a solution that is continuous in the closed region $0 \leqslant \rho \leqslant 1$. This can be verified, at least, for small values of the load parameter.

In view of the solution formula

$$\phi(\rho) = -\frac{1}{2}\int_0^1 \Phi(\rho, \xi)\frac{d}{d\xi}\left(\frac{d\psi}{d\xi}\right)^2 d\xi \tag{40}$$

a large portion of the behavior of the membrane stresses can be deduced without any explicit representation for the deflection function $\psi(\rho)$. (In the results to follow, we shall assume the load function $f(\rho)$ is one signed.) The reduced membrane stresses will be denoted by $\bar{N}^a$ or $\bar{N}^b$ with appropriate subscripts for the radial and tangential directions. As previously noted, the superscript a or b will indicate the nature of the boundary conditions on φ at $\rho = 1$.

Since by definition

$$\bar{N}_\rho = \frac{1}{\rho}\frac{d\phi}{d\rho} \tag{41}$$

the algebraic sign of $\bar{N}_\rho$ is carried by $d\phi/d\rho$ so that the tensile or compressive nature of $\bar{N}_\rho$ is determined by the algebraic sign of the integral

$$\frac{1}{2}\int_0^1 \frac{\partial^2\Phi}{\partial\rho\,\partial\xi}\left(\frac{d\psi}{d\xi}\right)^2 d\xi. \tag{42}$$

Now the function $\partial^2\Phi/\partial\rho\,\partial\xi$ for conditions (a) or (b) is a positive or zero function of its arguments on the range $0 \leqslant \xi \leqslant 1$, $0 \leqslant \rho \leqslant 1$. Hence we immediately have the fact that $\bar{N}_\rho{}^a, \bar{N}_\rho{}^b$ are tensile stresses throughout the plate. Furthermore

$$\frac{d\bar{N}_\rho}{d\rho} = -\frac{1}{2\rho^2}\int_0^1 \xi\left(\frac{d\psi}{d\xi}\right)^2 d\xi \tag{43}$$

and since the integral in (43) can never vanish for $0 < \rho \leqslant 1$, we have that the radial membrane stress steadily decreases as ρ increases. Thus the maximum value of $\bar{N}_\rho$ occurs at the center of the plate and has as its value

$$\begin{aligned}(\bar{N}_\rho{}^a)_{\max} &= \frac{1}{4}\int_0^1 \frac{(1-\xi^2)}{\xi}\left(\frac{d\psi}{d\xi}\right)^2 d\xi \\ (\bar{N}_\rho{}^b)_{\max} &= \frac{1}{4}\int_0^1 \frac{(1+\nu\xi^2)}{\xi}\left(\frac{d\psi}{d\xi}\right)^2 d\xi.\end{aligned} \tag{44}$$

Utilizing (43) again, we have (the compatibility equation)

$$\frac{d}{d\rho}(\bar{N}_\rho + \bar{N}_\theta) = -\frac{1}{2}\frac{1}{\rho}\left(\frac{d\psi}{d\rho}\right)^2 \tag{45}$$

and hence the sum of the principal stresses (membrane) is a non-increasing function so that it attains its maximum value at the origin. From the form of the right hand side of (45) it is evident that not much can be said about $\bar{N}_\theta$ until some specific assumptions are made concerning the behavior of $d\psi/d\rho$.

We now turn our attention to a consideration of the deflection function $\psi(\rho)$. An integration of the second of Eqs. 1 yields

$$\frac{d}{d\rho}\{\nabla^2\psi\} = 12(1-\sigma^2)\bar{N}_\rho \frac{d\psi}{d\rho} + \frac{\lambda}{\rho}\int_0^\rho \xi f(\xi)\, d\xi, \tag{46}$$

which is of course the equation of equilibrium in the vertical direction. If we now set $d\psi/d\rho = \Gamma$, we may write (46) as

$$\frac{d}{d\rho}\left\{\rho \frac{d\Gamma}{d\rho}\right\} - \left[12(1-\sigma^2)\rho\bar{N}_\rho + \frac{1}{\rho}\right]\Gamma = \lambda Q(\rho). \tag{47}$$

Since by the definition of $\bar{N}_\rho$ it is continuous on the range $0 \leqslant \rho \leqslant 1$ it will make sense to talk about the Green's function of

$$\frac{d}{d\rho}\left\{\rho \frac{d\Gamma}{d\rho}\right\} - \left[12(1-\sigma^2)\rho\bar{N}_\rho + \frac{1}{\rho}\right]\Gamma = 0. \tag{48}$$

To actually determine the Green's function, one would have had to solve the problem; however, we shall only be concerned with certain properties of this Green's function which we can deduce without a knowledge of $\bar{N}_\rho$. Multiplying through (47) by Γ and integrating by parts, we have

$$\Gamma\Gamma'\rho\Big]_0^1 - \int_0^1 \left\{\rho\Gamma'^2 + \left[12(1-\sigma^2)\rho\bar{N}_\rho + \frac{1}{\rho}\right]\Gamma^2\right\} d\rho = \lambda\int_0^1 Q(\rho)\Gamma(\rho)\, d\rho \tag{49}$$

but

$$\Gamma(\rho) = \lambda\int_0^1 G(\rho,\xi)Q(\xi)\, d\xi \tag{50}$$

hence

$$\Gamma\Gamma'\rho\Big]_0^1 - \int_0^1 \left\{\rho\Gamma'^2 + \left[12(1-\sigma^2)\rho\bar{N}_\rho + \frac{1}{\rho}\right]\Gamma^2\right\} d\rho = \lambda^2\int_0^1\int_0^1 G(\rho,\xi)Q(\rho)Q(\xi)\, d\rho\, d\xi. \tag{51}$$

Now the boundary term $\Gamma\Gamma'\rho$ is either negative or zero so that if $\Gamma \not\equiv 0$, then

$$\int_0^1 \int_0^1 G(\rho, \xi) Q(\rho) Q(\xi)\, d\rho\, d\xi < 0 \tag{52}$$

for any $Q(\rho) \not\equiv 0$. Consequently $G(\rho, \xi)$ is negative definite.

If we write $\bar{\Gamma}$ for the slope in the linear case and

$$E = \Gamma - \bar{\Gamma} \tag{53}$$

then we have the following differential equation for the excess function E

$$\frac{d}{d\rho}\left\{\rho \frac{dE}{d\rho}\right\} - \left[12(1-\sigma^2)\rho \bar{N}_\rho + \frac{1}{\rho}\right] E = \left[12(1-\sigma^2)\rho \bar{N}_\rho + \frac{1}{\rho}\right] \bar{\Gamma} \tag{54}$$

so that

$$E(\rho) = \int_0^1 G(\rho, \xi) \left[12(1-\sigma^2)\xi \bar{N}_\xi + \frac{1}{\xi}\right] \bar{\Gamma}\, d\xi. \tag{55}$$

Since $G(\rho, \xi) \leqslant 0$ for $\bar{N}_\xi \geqslant 0$ it follows that $\bar{\Gamma} \leqslant 0$ and we have consequently

$$E \geqslant 0. \tag{56}$$

But since $\Gamma \leqslant 0$, $\bar{\Gamma} \leqslant 0$ we have

$$|\bar{\Gamma}| \geqslant |\Gamma| \quad \text{for} \quad 0 \leqslant \rho \leqslant 1. \tag{57}$$

From this it follows that the maximum deflection is always attained at the center of the plate and that the linear value always exceeds the non-linear value by the amount

$$(\bar{\psi} - \psi)_{\rho=0} = \int_0^1 \int_0^1 G(\rho, \xi) \left[12(1-\sigma^2)\xi \bar{N}_\xi + \frac{1}{\xi}\right] \bar{\Gamma}(\xi)\, d\rho\, d\xi. \tag{58}$$

An appraisal of this term would yield some idea about the deflection at the center. Since $G(\rho, \xi)$ cannot be directly determined, we first need an estimation of it. We can obtain two Green's functions bounding $G(\rho, \xi)$ from above and below in the following fashion. Let $G_1(\rho, \xi)$, $G_2(\rho, \xi)$ be the Green's functions for

$$\begin{aligned} \frac{d}{d\rho}\left\{\rho \frac{dG_1}{d\rho}\right\} - \left[12(1-\sigma^2)\rho \bar{N}_\rho^1 + \frac{1}{\rho}\right] G_1 = 0 \\ \frac{d}{d\rho}\left\{\rho \frac{dG_2}{d\rho}\right\} - \left[12(1-\sigma^2)\rho \bar{N}_\rho^2 + \frac{1}{\rho}\right] G_2 = 0 \end{aligned} \tag{59}$$

respectively. Then

$$\int_0^1 G_2(\rho, \xi_2) \frac{d}{d\rho}\left\{\rho \frac{dG_1}{d\rho}\right\} d\rho - \int_0^1 G_1(\rho, \xi_1) \frac{d}{d\rho}\left\{\rho \frac{dG_2}{d\rho}\right\} = 12(1 - \sigma^2) \int_0^1 \rho(\bar{N}_\rho{}^1 - \bar{N}_\rho{}^2) G_1 G_2 \, d\rho \quad (60)$$

or

$$-G_2(\xi_1, \xi_2) + G_1(\xi_1, \xi_2) = 12(1 - \sigma^2) \int_0^1 \rho(\bar{N}_\rho{}^1 - \bar{N}_\rho{}^2) G_1(\rho, \xi_1) G_2(\rho, \xi_2) \, d\rho. \quad (61)$$

Now since G_1, G_2 are everywhere negative, we have that

$$|G_2(\xi_1, \xi_2)| > |G_1(\xi_1, \xi_2)| \qquad 0 \leqslant \xi_1, \xi_2 \leqslant 1 \quad (62)$$

provided $\bar{N}_\rho{}^1 - \bar{N}_\rho{}^2 \geqslant 0$ and $0 \leqslant \rho \leqslant 1$. Hence if a suitable numerical value can be found such that $12(1 - \sigma^2)\bar{N}_\rho \geqslant \mu^2$, then the Green's function of

$$\frac{d}{d\rho}\left\{\rho \frac{dG^*}{d\rho}\right\} - \left(\mu^2 \rho + \frac{1}{\rho}\right) G^* = 0 \quad (63)$$

will always be larger in absolute value than the true Green's function. Since the Green's function of (63) is tractable a suitable expression can always be found for G^* and we may write

$$|\bar{\psi} - \psi|_{\rho=0} \leqslant \int_0^1 \int_0^1 G^*(\rho, \xi)\left[12(1 - \sigma^2)\xi \bar{N}_\xi + \frac{1}{\xi}\right]\bar{\Gamma}(\xi) \, d\xi \, d\rho. \quad (64)$$

It may be noted here that for the assumptions we have made here, it can be readily shown that the slope function $\Gamma(\rho)$ and consequently the radial stress can be represented as alternating series in the load parameter. Consequently, the standard theorems concerning the error involved in truncating an alternating series enables one to obtain reasonably good upper and lower bounds for $\bar{N}_\rho$ which in turn permits estimations of $G(\rho, \xi)$.

We shall close our discussions here with an investigation of the error involved in truncating the system (13) after m equations. From a point of view of practice, this is of utmost importance, since it is impossible to solve the infinite set of equations (13). Consequently, some bound for the error of truncation will render justifiable the large computational task encountered in the solution of these non-linear problems.

Assuming that exact numerical results can be obtained for the m coefficients in the truncated expansion $\Gamma^m(\rho)$ then we have that $\Gamma^m(\rho)$ is a solution of the following integral equation:

$$\Gamma^m(\rho) = \lambda \bar{\Gamma}^m(\rho) + 12(1 - \sigma^2) \int_0^1 L(\rho, \xi) \xi \bar{N}_\xi{}^m \Gamma^m(\xi) \, d\xi \quad (65)$$

where $\bar{\Gamma}^m(\rho)$ represents the first m terms in the expansion for $\bar{\Gamma}$. By differentiation of (65) and some algebraic operations we obtain the result that $\Gamma^m(\rho)$ satisfies the following differential equation:

$$\mathfrak{L}^m(\Gamma^m) \equiv \frac{d}{d\rho}\left\{\rho \frac{d\Gamma^m}{d\rho}\right\} - \left[12(1-\sigma^2)\rho \bar{N}_\rho^m + \frac{1}{\rho}\right]\Gamma^m = \lambda Q^m(\rho) \quad (66)$$

plus whatever boundary conditions on $\Gamma^m(\rho)$ are necessary. If $\bar{N}_\rho^m$ were known (by definition it is continuous on $[0, 1]$ since

$$\bar{N}_\rho^m = \frac{1}{2\rho}\int_0^1 \frac{\partial^2 \Phi}{\partial \rho\, \partial \xi}(\Gamma^m)^2\, d\xi \quad (67)$$

then one could compute the Green's function for

$$\mathfrak{L}^m(G^m) = 0 \quad (68)$$

plus appropriate boundary conditions. With this function we have directly

$$\Gamma^m(\rho) = \lambda \int_0^1 G^m(\rho, \xi) Q^m(\xi)\, d\xi. \quad (69)$$

Now since $Q^m(\xi)$ represents the first m terms in the expansion of a function in a generalized Fourier series, it is possible to find an m such that

$$Q^m(\xi) = Q(\xi) + \theta_m(\xi) \quad \text{where} \quad |\theta_m(\xi)| \leqslant \epsilon \quad (70)$$

almost everywhere on the unit interval. Hence we may write

$$|\Gamma^m(\rho)| \leqslant -\lambda \int_0^1 G_L(\rho, \xi) Q(\xi)\, d\xi + \lambda \bar{\theta}_m(\xi) \quad |\bar{\theta}_m(\xi)| \leqslant \epsilon. \quad (71)$$

Consequently for sufficiently large m, we have that

$$|\Gamma^m(\rho)| \leqslant |\bar{\Gamma}(\rho)| + \epsilon_2(\lambda, m). \quad (72)$$

The difference between Γ and Γ^m can be written by utilizing the Green's function for (48) as

$$\Gamma - \Gamma^m = \lambda \int_0^1 [Q(\xi) - Q^m(\xi)] G(\rho, \xi)\, d\xi + 12(1-\sigma^2) \int_0^1 \xi(\bar{N}_\xi - \bar{N}_\xi^m)\Gamma^m G(\rho, \xi)\, d\xi. \quad (73)$$

Now we have that

$$|\xi \bar{N}_\xi - \xi \bar{N}_\xi^m| \leqslant \tfrac{1}{2}|\Gamma - \Gamma^m|_{\max} \int_0^1 \left|\frac{\partial^2 \Phi}{\partial \rho\, \partial \xi}\right| |\Gamma + \Gamma^m|\, d\xi. \quad (74)$$

Consequently, the absolute value of the truncation error is estimated by

$$|\Gamma - \Gamma^m| \leqslant -\lambda \int_0^1 |Q - Q^m| G \, d\xi + 6(1 - \sigma^2) |\Gamma - \Gamma^m|_{\max} \int_0^1 \int_0^1 \frac{\partial^2 \Phi}{\partial \rho \, \partial \xi} |\Gamma + \Gamma^m| \Gamma^m G \, d\xi. \quad (75)$$

In view of (72), the coefficient of $|\Gamma - \Gamma^m|_{\max}$ can be made less than one in absolute value by choosing m large enough for suitable λ. So that we have

$$|\Gamma - \Gamma^m| \leqslant \lambda F(\rho, m) + \alpha |\Gamma - \Gamma^m|_{\max}. \quad (76)$$

Since $\Gamma - \Gamma^m$ is continuous on $[0, 1]$, it attains its maximum value and we have

$$|\Gamma - \Gamma^m|_{\max} \leqslant \frac{\lambda F(\bar{\rho}, m)}{1 - \alpha}. \quad (77)$$

Since $\alpha < \kappa \leqslant 1$ and $F(\bar{\rho}, m)$ goes to zero with increasing m, we have that the approximants Γ^m actually yield the solution of (7) in the limit. Furthermore this provides uniqueness of the solution of (7) for suitable λ.

Sine-Cosine Potentiometer. — A miniature precision sine-cosine potentiometer which will operate on direct current has been developed by Computer Instruments Company, Brooklyn, and is now available for industrial use. Designated the type 106, it is a resistance device, which works on the principle that the wiper voltage is proportional to resistance. Therefore, unlike magnetic resolvers, which also perform the sine-cosine function but can give out only a-c. voltages, it will operate on either alternating or direct current without booster or compensating amplifiers.

The type 106 is the smallest packaged potentiometer ever produced which will perform the sine-cosine function at high impedance levels and without loss of resolution. It measures $1\frac{3}{32}$ in. in diameter by $\frac{1}{16}$ in. in length and weighs only one ounce. It can be operated at extremely high shaft velocities without producing phase distortions or amplitude modulations. Because of its small size and great accuracy, it is now being used in electronic computer systems designed for gunfire control, navigation and other automatic controls.

The performance of the sinusoidal function is accomplished by a combination of mechanical-shaping and potential-field-distribution techniques applied to the carbon film. The transition from zero to finite slopes is achieved without abrupt steps in output. Both sine and cosine outputs are produced simultaneously by means of two electrically independent wipers spaced 90° apart. If two sine and two cosine outputs are desired, four electrically independent wipers can be used, also spaced 90° apart.

Two models of the type 106 are being produced. The standard type 106 is equipped with double ball bearings and has a flange-type servo mounting. Two or more potentiometers can be ganged in factory-assembled units without loss of mechanical or electrical accuracy. Ganged units have no backlash since they have a common shaft, which may be double-ended.

A modification of the standard model is the type 106J, which is equipped with a sleeve bearing and may have either a bushing or flange-type mounting. Shaft-locking bushings, knob flats and screwdriver slots are available. Shaft extensions are limited to one end only. With the type 106J, ganging is restricted to two sections, which are factory assembled and phased.

Hardwood Floors for Trucks.—A special hardwood floor overlay, designed at the request of the Government to restore smoothness to steel cargo truck body floors, has been developed under the nation's hardwood research program at the laboratory of Timber Engineering Company, research affiliate of National Lumber Manufacturers Association.

The new floor, installed in a 2½-ton Army truck, has started its longtime, rigorous loading tests at the Army's Aberdeen, Maryland, Proving Ground.

Made of red oak lumber, edge-glued with waterproof marine adhesive, the hardwood floor is treated preservatively and for dimensional stabilization. It was developed as a solution to the military's problems presented by steel floors becoming dented into a "washboard" surface that makes cargo loading and handling difficult.

The hardwood floor is designed for installation over the steel floors in the Army's present trucks. It not only restores a level surface to the truck body's floor, but provides a more comfortable floor for personnel using it in any climatic extremes.

Successful performance of the wood floor throughout the Aberdeen tests is expected to result in its widespread installation in military and commercial vehicles.

NONLINEAR STUDY OF FREQUENCY MODULATED OSCILLATOR *

BY

JOSEF GROSS [1]

INTRODUCTION

The linear formulation of the problem of a frequency modulated oscillator leads to the following fallacies:

Substitution of a linear non-dissipative circuit (where undamped oscillations must subsist) for a self-sustained oscillatory system is in principle inadmissible, since it leads here to an infinity of bands of parametric excitation (characterized by a relation between the modulating frequency p and the carrier frequency ω_0) where the circuit attains no steady state. If dissipation is taken into account, the number of bands of parametric excitation (characterized now by p, ω_0, and the circuit damping) is finite, but no undamped oscillation occurs.

In the following it is shown how the nonlinear damping of the tube limits the growth of oscillations in the case of parametric excitation, and thus permits the establishment of a stationary oscillation.

LINEAR CIRCUIT WITH PERIODICALLY VARYING CAPACITANCE

A frequency modulated triode oscillator has in several previous papers been idealized as a linear non-dissipative parallel tuned circuit with periodically varying capacitance (1).[2] In such a circuit the charge x on the capacitor obeys

$$\frac{dx}{dt} + \frac{1}{L}\int_0^t \frac{x}{C}\,dt = 0, \tag{1}$$

or

$$\frac{d}{dt}\left(L\frac{dx}{dt}\right) + \frac{x}{C} = 0. \tag{2}$$

If the plates of a plane capacitor

$$C = \frac{S\epsilon}{s}, \tag{3}$$

where S is the area, s the distance between the plates, ϵ the dielectric

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

const., are moved according to

$$s = s_0(1 + \gamma \cos pt), \tag{4}$$

the weight function in (2) is simple harmonic with period $2\pi/p$, yielding for x the Mathieu equation

$$\ddot{x} + \omega_0^2(1 + \gamma \cos pt)x = 0, \tag{5}$$

where

$$\omega_0^2 = \frac{1}{LC_0}, \quad C_0 = \frac{S\epsilon}{s_0}.$$

With $pt = 2z$, and $'$ denoting differentiation with respect to z, we have

$$x'' + [a - 2q \cos 2z]x = 0 \tag{6}$$

in the canonical form, where

$$a = 4\left(\frac{\omega_0}{p}\right)^2, \quad q = -2\gamma\left(\frac{\omega_0}{p}\right)^2.$$

The solution of (6) is

$$x = Ae^{\mu z}\phi(z) + Be^{-\mu z}\phi(-z) \quad \text{for} \quad \mu \neq jm, \quad m = 0, 1, \cdots \tag{7}$$

with $\phi(z)$ periodic with period π or 2π. The value of μ as a function of a, q is plotted in the "stability chart" (2, p. 98). Imaginary values of μ correspond to stable solutions,[3] real values to unstable solutions. For $\gamma = \text{const.} \ll 1$, the unstable regions occur around $a = n^2(n = 1, 2 \cdots)$, that is $p = 2\omega_0/n$. From the chart it is evident that the greatest instability (value of $|\mu|$, μ real) occurs at $a = 1$, that is, $p = 2\omega_0$ (there, for $\gamma \ll 1$, $\mu \doteq \frac{\gamma}{4}$, corresponding to a negative resistance of magnitude $\frac{\gamma}{2}\sqrt{\frac{L}{C_0}}$). The instability decreases with $\frac{p}{\omega_0}$.

Thus the non-dissipative linear circuit (1) exhibits an infinite number of bands of parametric excitation (where the circuit attains no steady state), which, for $\gamma = \text{const.} \ll 1$, decrease in width and instability as $\frac{p}{\omega_0}$ decreases.

Assumption of a linear dissipative circuit merely results in a finite number of bands of parametric excitation, characterized now by the requirement (see Appendix I)

$$\mu p > \delta. \tag{8}$$

The steady state which is achieved in a frequency-modulated triode

[3] A solution is stable if it is bounded for all $z \geq 0$, unstable if unbounded. Not to be confused with stability (in the sense of Lyapunov) of periodic solutions.

oscillator for $p \cong \frac{2}{n}\omega_0$, $n = 1, 2, \cdots$ can be accounted for only by the nonlinear damping of the triode. In the following, the case $p = 2\omega_0$ will be examined, and shown to be stable; *a fortiori*, in practical frequency modulated oscillators, $\frac{p}{\omega_0} \ll 1$, no instability occurs.

NONLINEAR CIRCUIT WITH PERIODICALLY VARYING CAPACITANCE

The current x through the inductance of the tank circuit in a tuned plate frequency modulated oscillator $\left[C(t) = \frac{C_0}{1 + \gamma \cos pt}\right]$ obeys, to $0(\gamma^2)$ (see Appendix II)

$$x'' + [c + 2\gamma \sin 2z]x' + (1 + \gamma \cos 2z)x + ax'^3 = 0. \tag{9}$$

Here

$$c = \frac{\delta - \beta}{\omega_0}, \quad a = \alpha\omega_0, \quad \delta = \frac{R}{L}, \quad \beta = g_m M\omega_0^2, \quad \alpha = \tfrac{1}{3}\beta\left(\frac{M}{V}\right)^2.$$

This equation is derived for shallow and very rapid modulation: $\gamma \ll 1$, $p = 2\omega_0$, on the assumption of a slightly damped tank circuit with loose feedback coupling: $c = 0(\gamma)$, $a = 0(\gamma)$. To make the orders of magnitude more distinct, rewrite

$$x'' + x + \gamma\{[C + 2 \sin 2z]x' + \cos 2z\, x + Ax'^3\} = 0 \tag{10}$$

where $C = \frac{c}{\gamma}$, $A = \frac{a}{\gamma}$ are finite constants of order 1.

Equation 10 is a quasi-harmonic, non-autonomous system of the form $x'' + x + \gamma f(x, x', z) = 0$. The first approximation derived in Appendix III seeks a solution of (10) in the form

$$x = r \cos(\varphi - z), \quad \text{with} \quad x' = r \sin(\varphi - z), \tag{11}$$

where the slowly varying amplitude and phase obey

$$\left.\begin{aligned} \frac{dr}{dz} &= -\frac{\gamma}{2\pi}\int_0^{2\pi} f[r\cos(\varphi - z), \\ &\qquad r\sin(\varphi - z), z]\sin(\varphi - z)\,dz = \gamma P(r, \varphi) \\ \frac{d\varphi}{dz} &= -\frac{\gamma}{2\pi r}\int_0^{2\pi} f[r\cos(\varphi - z), \\ &\qquad r\sin(\varphi - z), z]\cos(\varphi - z)\,dz = \gamma Q(r, \varphi) \end{aligned}\right\}. \tag{12}$$

A singular point (r_0, φ_0) of (12) corresponds to a stationary periodic solution of (9), close to the harmonic motion $x = r_0 \cos(\varphi_0 - z)$; stability

of the stationary periodic solution is inferred from the stability properties of the corresponding singular point.

For the nonlinear function in (10) the "abbreviated" system is

$$\left.\begin{aligned} \frac{dr}{dz} &= \gamma \frac{r}{2}\left[\tfrac{1}{2}\sin 2\varphi - C - \tfrac{3}{4}Ar^2\right] \\ \frac{d\varphi}{dz} &= \frac{\gamma}{4}\left[\cos 2\varphi\right] \end{aligned}\right\}. \tag{13}$$

The nature of the singular points is easily determined here, since Q is a function of φ only. Hence the behavior of the solutions of (13″) is determined by the location and character of its singularities $Q(\varphi) = 0$. Thus $\varphi_0 = \frac{\pi}{4}$ is a stable phase to which any other initial phase tends with increasing z $\left[\text{since there } \frac{d\varphi}{dz} = 0, \text{ and } \frac{d}{d\varphi}\frac{d\varphi}{dz} < 0\right]$. Once this phase has been attained, the amplitude obeys

$$\frac{dr}{dz} = \gamma \frac{r}{2}\left[\tfrac{1}{2} - C - \tfrac{3}{4}Ar^2\right] = \gamma P\left(r, \frac{\pi}{4}\right). \tag{14}$$

Of the two stationary amplitudes

(i) $r_0 = 0$ is stable for $C > \frac{1}{2}$, unstable for $C < \frac{1}{2}$;

(ii) $r_0^2 = \frac{4}{3A}\left(\frac{1}{2} - C\right)$ is unstable for $C > \frac{1}{2}$, stable for $C < \frac{1}{2}$.

(Evidently a physically possible non-zero value of r_0^2 has to be positive.)

The quasi-harmonic system (9), which describes parametric excitation in a triode oscillator, has the following behavior:

$c < \frac{\gamma}{2}$—the system is self-excited and has a stationary solution

$$x \cong \sqrt{\frac{2\gamma - 4c}{3a}} \cos\left(\frac{\pi}{4} - z\right), \tag{15}$$

$c > \frac{\gamma}{2}$—the system is not self-excited, its rest position is stable.

In contrast, for the unmodulated triode oscillator, governed by (9) with $\gamma = 0$

$$x'' + x + cx' + ax'^3 = 0, \tag{16}$$

application of the averaging principle (Eq. 12 of Appendix III) leads to

$$\left.\begin{aligned}\frac{dr}{dz} &= -\frac{r}{2}\left[c + \tfrac{3}{4}ar^2\right] \\ \frac{d\varphi}{dz} &= 0\end{aligned}\right\}. \tag{17}$$

Hence (16) is self-excited for $c < 0$ [that is, $\delta < \beta$, effective damping at rest negative, making the rest position unstable] with stationary solution

$$x \cong \sqrt{\frac{-4c}{3a}} \cos(z + \lambda), \quad \lambda \text{ arbitrary}, \tag{18}$$

or it is stable at rest for $c > 0$.

Thus the modulated oscillator is able to overcome, through the negative damping of parametric excitation (negative resistance of magnitude $\frac{\gamma}{2}\sqrt{\frac{L}{C_0}}$), a positive damping at rest up to $c = \frac{\gamma}{2}$; the stationary amplitude for $c > 0$ is then correspondingly reduced.

APPENDIX I. PARAMETRIC EXCITATION IN A DISSIPATIVE LINEAR CIRCUIT

The dissipative linear circuit with periodically varying capacitance is described by

$$\ddot{x} + \delta\dot{x} + \omega_0^2(1 + \gamma \cos pt)x = 0, \tag{5'}$$

where $\delta = \frac{R}{L}$. Let $x = ye^{-\frac{\delta t}{2}}$, then y obeys

$$\ddot{y} + (\omega_1^2 + \gamma\omega_0^2 \cos pt)y = 0,$$

where $\omega_1^2 = \omega_0^2 - \frac{\delta^2}{4}$, and $\frac{\delta}{\omega_1} \ll 1$. With $pt = 2z$

$$y'' + [a' - 2q' \cos 2z]y = 0, \tag{6'}$$

where $a' = 4\left(\frac{\omega_1}{p}\right)^2$, $q' = -2\gamma\left(\frac{\omega_0}{p}\right)^2$. The solution of (6') is, for $\mu \neq jm$, $m = 0, 1, \cdots$

$$y = Ae^{\mu z}\phi(z) + Be^{-\mu z}\phi(-z) \tag{7'}$$

with ϕ periodic with period π or 2π, and hence the solution of (5')

$$x = Ae^{\frac{1}{2}(\mu p - \delta)t}\phi(\tfrac{1}{2}pt) + Be^{-\frac{1}{2}(\mu p + \delta)t}\phi(-\tfrac{1}{2}pt).$$

Parametric excitation occurs for $\mu p > \delta$, which for $\gamma \ll 1$ can be met only for $\frac{p}{\omega_0} > N$, where $N = N(\delta)$, since μ decreases with $\frac{p}{\omega_0}$. Again the insta-

bility is greatest for $a' = 1$, that is, $p = 2\omega_1$, and there, for $\gamma \ll 1$, $\mu \doteq \frac{\gamma}{4}\left(\frac{\omega_0}{\omega_1}\right)^2$ (2, p. 101), and the condition for parametric excitation is $\gamma > 2\frac{\delta}{\omega_1}$.

The mechanism of parametric excitation can be explained intuitively for the case $p = 2\omega_1$: The force required to move the plates of the capacitor (3) according to (4) against the electrostatic field is

$$F = \frac{dW}{ds} = \frac{x^2}{2S\epsilon}$$

where $W = \frac{x^2}{2C}$. For $\gamma \ll 1$, the charge is approximately (over one cycle)

$$x \doteq A \cos(\omega_1 t + \varphi).$$

The required work input per modulation cycle is then

$$W_i = \oint F ds = -\gamma \frac{A^2}{4C_0} \int_0^{2\pi} [1 + \cos 2(\omega_1 t + \varphi)] \sin pt \, dpt.$$

With the approximate form of the solution assumed, the integral vanishes, unless $p = 2\omega_1$; then $W_i = \gamma \frac{A^2\pi}{4C_0} \sin 2\varphi$. The parametric oscillation will occur at a phase (relative to the parameter variation) which leads to maximum energy absorption by the circuit

$$\varphi = \frac{\pi}{4}, \quad W_i = \gamma \frac{A^2\pi}{4C_0}.$$

The energy dissipated in R in the same time interval is

$$W_d = \int_0^{2\pi/p} \dot{x}^2 R \, dt = \omega_1^2 R A^2 \int_0^{2\pi/p} \sin^2(\omega_1 t + \varphi) \, dt = RA^2\omega_1^2 \frac{\pi}{p}.$$

The condition for parametric excitation is $W_i \geq W_d$, or $\gamma \geq 2\frac{\delta}{\omega_1}$.

APPENDIX II. DERIVATION OF THE DIFFERENTIAL EQUATION FOR PARAMETRIC EXCITATION IN A TRIODE OSCILLATOR

The tuned plate triode oscillator (Fig. 1), whose tuning capacitance is varied in accordance with

$$C(t) = C_0 g(t), \quad g(t) = \frac{1}{1 + \gamma \cos pt}, \tag{1}$$

is treated under the following assumptions:

(i) Class A operation (that is, $v_g \leq V$)
(ii) No grid current
(iii) $r_p \to \infty$ (that is, $\mu \to \infty$).

The tube transfer characteristic relating the alternating components of plate current (i_p) and grid voltage (v_g) is expressed as

$$i_p = g_m v_g \left[1 - \frac{1}{3}\frac{v_g^2}{V^2}\right] \tag{2}$$

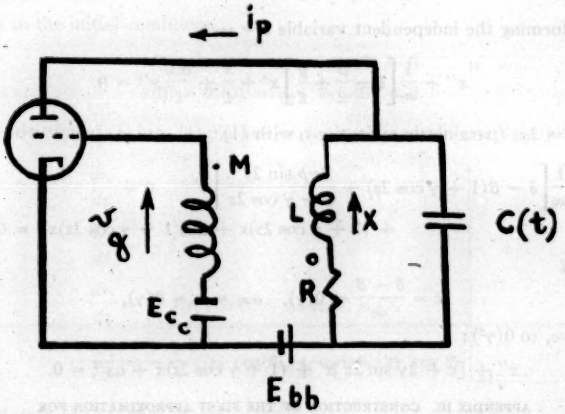

FIG. 1.

where g_m is the slope of the transfer characteristic at the operating point, and V is the maximum allowable grid voltage excursion (saturation voltage).

The alternating current x in the inductance L obeys:

$$L\dot{x} + Rx = \frac{1}{C}\int_0^t (i_p - x)\, dt. \tag{3}$$

Differentiating, and with $v_g = M\dot{x}$, and (2):

$$C(L\ddot{x} + R\dot{x}) + \dot{C}(L\dot{x} + Rx) + x = i_p = g_m M\dot{x}\left[1 - \frac{1}{3}\left(\frac{M\dot{x}}{V}\right)^2\right]. \tag{4}$$

Let

$$\delta = \frac{R}{L}, \quad \omega_0^2 = \frac{1}{LC_0}, \quad \beta = g_m M\omega_0^2, \quad \alpha = \tfrac{1}{3}\beta\left(\frac{M}{V}\right)^2;$$

then

$$\ddot{x} + \left[\delta - \frac{\beta}{g} + \frac{\dot{g}}{g}\right]\dot{x} + \frac{\delta\dot{g} + \omega_0^2}{g}x + \frac{\alpha}{g}\dot{x}^3 = 0. \tag{5}$$

For the case of a slightly damped tank circuit $\frac{\delta}{\omega_0} = 0(\gamma)$, and shallow modula-

tion $\gamma \ll 1$, with loose feedback coupling $\alpha\omega_0 = 0(\gamma)$ and certain self-excitation $\frac{\beta}{\omega_0} = 0(\gamma)$, we have, to $0(\gamma^2)$:

$$\ddot{x} + \left[\delta - \frac{\beta}{g} + \frac{\dot{g}}{g}\right]\dot{x} + \frac{\omega_0^2}{g}x + \frac{\alpha}{g}\dot{x}^3 = 0. \tag{6}$$

Transforming the independent variable $z = \omega_0 t$:

$$x'' + \frac{1}{\omega_0}\left[\delta - \frac{\beta}{g} + \frac{\dot{g}}{g}\right]x' + \frac{x}{g} + \frac{\alpha\omega_0}{g}x'^3 = 0. \tag{7}$$

For $p = 2\omega_0$ (parametric excitation), with (1):

$$x'' + \frac{1}{\omega_0}\left[\delta - \beta(1 + \gamma\cos 2z) + \frac{\gamma p \sin 2z}{1 + \gamma\cos 2z}\right]x' + (1 + \gamma\cos 2z)x + \alpha\omega_0(1 + \gamma\cos 2z)x'^3 = 0. \tag{8}$$

Calling

$$c = \frac{\delta - \beta}{\omega_0} = 0(\gamma), \quad \alpha\omega_0 = a = 0(\gamma),$$

we have, to $0(\gamma^2)$:

$$x'' + [c + 2\gamma\sin 2z]x' + (1 + \gamma\cos 2z)x + ax'^3 = 0. \tag{9}$$

APPENDIX III. CONSTRUCTION OF THE FIRST APPROXIMATION FOR NON-AUTONOMOUS SYSTEMS[4]

The differential equation

$$x'' + x + \gamma f(x, x', z) = 0 \tag{1}$$

where f is a polynomial in x, x', $\sin 2z$, $\cos 2z$, is called, for $\gamma \ll 1$, quasiharmonic. We seek a periodic solution, close to the harmonic motion resulting from (1) with $\gamma = 0$. The equivalent system of first order differential equations is:

$$\left.\begin{aligned} y &= x' \\ y' &= -x - \gamma f(x, x', z) \end{aligned}\right\}. \tag{2}$$

A change of variables

$$\left.\begin{aligned} x &= r\cos\theta \quad r^2 = x^2 + y^2 \\ y &= r\sin\theta \quad \theta = tg^{-1}\frac{y}{x} \end{aligned}\right\} \tag{3}$$

transforms (2) into:

$$\left.\begin{aligned} \frac{dr}{dz} &= -\gamma f(r\cos\theta, r\sin\theta, z)\sin\theta \\ \frac{d\theta}{dz} &= -1 - \frac{\gamma}{r}f(r\cos\theta, r\sin\theta, z)\cos\theta. \end{aligned}\right\} \tag{4}$$

[4] The following derivation is a modification of that given by Minorsky (3, 4).

We assume a formal solution to the above:

$$\left.\begin{aligned} r(z) &= \sum_{n=0} \gamma^n r_n(z) \\ \theta(z) &= \sum_{n=0} \gamma^n \theta_n(z) \end{aligned}\right\} \tag{5}$$

subject to the initial conditions

$$\begin{aligned} r(0) &= r_0(0) = r_0 \qquad r_{n>0}(0) = 0 \\ \theta(0) &= \theta_0(0) = \varphi_0 \qquad \theta_{n>0}(0) = 0. \end{aligned} \tag{6}$$

Substitution of (5) into (4) yields a sequence of linear differential equations:

$$\left.\begin{aligned} \frac{dr_0}{dz} &= 0 \\ \frac{d\theta_0}{dz} &= -1 \end{aligned}\right\} \tag{7_1}$$

$$\left.\begin{aligned} \frac{dr_1}{dz} &= -f(r_0 \cos\theta_0, r_0 \sin\theta_0, z) \sin\theta_0 \\ \frac{d\theta_1}{dz} &= -\frac{1}{r_0} f(r_0 \cos\theta_0, r_0 \sin\theta_0, z) \cos\theta_0, \end{aligned}\right\} \text{etc.} \tag{7_2}$$

The solution of (7_1), with (6), is

$$\left.\begin{aligned} r_0(z) &= r_0 \\ \theta_0(z) &= \varphi_0 - z \end{aligned}\right\}. \tag{8}$$

Hence the differential equation of the first approximation (Eq. 4 to $0(\gamma^2)$) is:

$$\left.\begin{aligned} \frac{dr}{dz} &= -\gamma f[r_0 \cos(\varphi_0 - z), r_0 \sin(\varphi_0 - z), z] \sin(\varphi_0 - z) \\ \frac{d\theta}{dz} &= -1 - \frac{\gamma}{r_0} f[r_0 \cos(\varphi_0 - z), r_0 \sin(\varphi_0 - z), z] \cos(\varphi_0 - z) \end{aligned}\right\}. \tag{9}$$

For a study of the asymptotic behavior of the solutions of (9), the right hand sides are replaced by their "slow" components, yielding the "abbreviated" system of the first approximation:

$$\left.\begin{aligned} \frac{dr}{dz} &= -\frac{\gamma}{2\pi} \int_0^{2\pi} f[r \cos(\varphi - z), r \sin(\varphi - z), z] \sin(\varphi - z)\, dz = \gamma P(r, \varphi) \\ \frac{d\varphi}{dz} &= -\frac{\gamma}{2\pi r} \int_0^{2\pi} f[r \cos(\varphi - z), r \sin(\varphi - z), z] \cos(\varphi - z)\, dz = \gamma Q(r, \varphi) \end{aligned}\right\} \tag{10}$$

where r, φ denote the (previously assumed const.: (8)) "slow" components of r, θ. In evaluating the integrals, r, φ in the integrands are regarded as const.

It has been proved that the solutions of the autonomous system (10) approximate closely the solutions of (9) over large z intervals, provided γ is sufficiently small (5, p. 341). The system (10) is of the form studied by Poincaré; its singularities correspond to stationary periodic solutions of (9) [and hence of (1) to $0(\gamma^2)$], close to the harmonic motion

$$x = r_0 \cos(\varphi_0 - z) \tag{11}$$

where r_0, φ_0 are solutions of $P(r, \varphi) = Q(r, \varphi) = 0$. The stability of the stationary solution (11) is deduced from the character of the corresponding singular point of (10). Criteria for the stability of singular points when P and Q contain both r and φ are given in chapter V of Ref. 5. In case Q is a function of φ only (or P of r only), the "abbreviated" system splits up into two uncoupled differential equations of the form $dy/dz = f(y)$; the singular points y_i of such a system $f(y_i) = 0$ correspond to states of equilibrium which are stable if $f'(y_i) < 0$, unstable if > 0.

In case $f(x, x')$ does not contain z explicitly (autonomous system), the "abbreviated" system of the first approximation reduces to

$$\left.\begin{aligned} \frac{dr}{dz} &= -\frac{\gamma}{2\pi}\int_0^{2\pi} f[r\cos u, r\sin u]\sin u\, du = \gamma P(r) \\ \frac{d\varphi}{dz} &= \frac{\gamma}{2\pi r}\int_0^{2\pi} f[r\cos u, r\sin u]\cos u\, du = \gamma Q(r), \end{aligned}\right\} \tag{12}$$

which is the result stated on page 12 of Ref. 6.

REFERENCES

(1) EVERARD M. WILLIAMS AND L. VALLESE, "Wide Deviation Frequency-Modulated Oscillator," *Proc. I.R.E.*, Vol. 36, No. 10, p. 1282 (1948).

(2) N. W. MCLACHLAN, "Theory and Application of Mathieu Functions," Oxford, Clarendon Press, 1947.

(3) N. MINORSKY, "Parametric Excitation," *Jour. Appl. Phys.*, Vol. 22, No. 1, p. 49 (1951).

(4) N. MINORSKY, "Stationary Solutions of Certain Nonlinear Differential Equations," JOUR. FRANKLIN INST., Vol. 254, No. 1, p. 21 (1952).

(5) A. A. ANDRONOW AND C. E. CHAIKIN, "Theory of Oscillations," Princeton, Princeton University Press, 1949.

(6) N. KRYLOFF AND N. BOGOLIUBOFF, "Introduction to Nonlinear Mechanics," Princeton, Princeton University Press, 1943.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

A SHAFT DISPLACEMENT INDICATOR

A non-contacting distance gage that measures the relative position of a high-speed rotating shaft has been developed by L. A. Marzetta of the National Bureau of Standards at the request of the Department of Defense. The device employs a mutual inductance-type gage to indicate the longitudinal movement of the shaft, such as might be caused by failure or wear of a thrust bearing. The equipment is designed to give continuous dependable service over a long period of time.

In the field of heavy rotating machinery such as steam turbines and internal combustion engines, it is sometimes desirable to monitor the relative position of some moving member. Turbines, in particular, having certain types of load are subject to rapid, unpredictable thrust bearing wear. Since the rotor and stator of a turbine are machined to close tolerances, excess longitudinal movement of the shaft presents the danger of interference of these parts and possible destruction of the machine. The NBS shaft position indicator detects and measures accurately the shaft displacement in either direction. When the displacement exceeds a prescribed limit, the indicator actuates an alarm, thereby warning operating personnel of imminent machine failure. Since the instrument also gives a direct indication of the amount of bearing wear, a bearing may now be used for its full useful life and need not be replaced unnecessarily soon.

The detecting element in this system is a mutual inductance micrometer[1] developed by M. L. Greenough of the National Bureau of Standards. A transducer probe, consisting of two coaxial, coplanar coils wound on a dielectric core about one-half inch in diameter, is mounted on the turbine frame near the shaft. When the primary coil is energized from a regulated r–f source, a voltage is induced in the secondary coil. On the end of the turbine shaft is fastened a brass disk which forms part of the electrical system. Motion of the disk toward or away from the probe changes the mutual inductance between the two windings. Thus, the output voltage from the secondary is dependent on the spacing between the probe face and the brass disk. This voltage is indicated on a meter calibrated in thousandths of an inch and shows the shaft position relative to the main frame of the turbine. The probe assembly includes a graduated micrometer that provides for accurate adjustment of the spacing between the probe face and the disk.

* Communicated by the Director.

[1] "Technical Details of an Electronic Micrometer," *Electronics*, 20, 11 (November 1947).

Coupled to the micrometer probe is a 5-tube exciter unit which contains a regulated carrier oscillator to furnish current for the probe transducer, the detector circuit, and the meter for indicating the shaft position. In a separate chassis are the power supplies and the alarm circuits.

Although the instrument's function is to measure displacement of a rotating shaft in a longitudinal direction, it can easily be adapted to measure motion in the radial direction. Moreover, it can be applied to indicate the relative positions of oscillating, reciprocating, or quasi-stationary members over a wide range of displacements since the mutual inductance micrometer will measure lengths as small as 50 microinches or as large as several inches with an accuracy of 3 per cent.

SCREW-THREAD PITCH MEASUREMENT

A technique recently developed by C. E. Haven of the National Bureau of Standards permits rapid measurement of the pitch, or lead, of internal screw threads without sacrifice of the high accuracy required in this work. The NBS method can be used to determine taper of ring gages at the same time as lead and is adaptable to measurements in very small ring gages, which have hitherto presented a considerable problem. For straight-thread ring gages, the technique has the added advantage that straightness of the bore can be determined simultaneously with lead, thus providing a quick assessment of the amount of wear at the ends of the gage.

In connection with the program of the American Petroleum Institute for standardization of the threaded connections on oil field equipment, the National Bureau of Standards calibrates master thread gages used by the petroleum industry for calibration of working standards. This is done through the use of thread measuring instruments whose calibration is ultimately based on the national standard of length maintained by the Bureau. Because the standardized API threaded joints cover such an extensive range of sizes—$\frac{1}{8}$ to 20 in. in diameter and up to 6 in. of threaded length—the measurement of some elements of the threads is beyond the range of commercially available measuring equipment, and the National Bureau of Standards and the manufacturers of API gages have had to design special equipment for this purpose. Until now, NBS equipment for determination of lead has required the use of two series of equal combinations of gage blocks for measurement of the intervals, and the procedure involved has been rather time-consuming.

The more rapid method developed by NBS for measuring lead makes use of a taper testing machine constructed by Pratt and Whitney Division of Niles-Bement-Pond Co. This device consists primarily of an adjustable sine plate on which the gage to be tested is mounted, and a carriage which travels on accurately machined ways to move a ball-point probe in and out of the mounted gage. When the angle between

the sine plate and a normal to the plane of the ways is equal to one-half the taper angle of the gage under test, the pitch line at the bottom of the gage (provided the taper of the gage is straight) will be parallel to the ways. As the carriage moves along the ways, the small ball on the end of the probe contacts the flanks of the threads in the ring gage. If the taper of the gage is true, the ball will remain at the same elevation above the ways as the probe moves from thread to thread. Variations in the elevation of the probe, caused by errors in taper, are read on an electrically actuated meter.

The National Bureau of Standards has extended this procedure to permit use of the same settings of the probe to determine lead as well as taper. The NBS method for lead determination involves measurement of the movement of the carriage through the use of a glass scale graduated in increments of 0.050 in. over a range of 4 in. The glass scale moves with the carriage, and the position of the scale is observed with a microscope having four reticle graduations, each equivalent to 0.01 in. on the scale. A scale graduation is brought into apparent coincidence with the nearest reticle graduation by rotating a plane parallel plate of optical glass in the optical path of the microscope.[1] This parallel plate constitutes a kind of "optical micrometer," that is, the shaft on which it rotates carries a graduated drum having 200 divisions for each 0.01 in. of apparent motion of a scale graduation. This gives a least reading of 0.00005 in., and permits estimation of fifths of divisions. The scale is accurate to ± 0.00002 in. in one inch and ± 0.00004 in. in four inches.

For accurate measurement of lead, the motion of the carriage must be substantially free from friction. This is accomplished by having the carriage roll on $\frac{1}{4}$-in. steel balls located in suitable guides. The lower surfaces of these guides are formed by two parallel V-grooves, approximately $3\frac{1}{4}$ in. apart, running along the entire upper surface of the base on which the carriage travels. The corresponding upper surfaces of the two guides are formed by a single V-groove and a flat surface which run along the under side of the carriage. By viewing a mirror mounted on the carriage with an autocollimating telescope, it is possible to determine deviations from linear motion of the carriage quite accurately and to make such adjustments of the ways as are necessary to obtain practically straight-line motion of the carriage.

The modified instrument has proved highly satisfactory at NBS for rapid, accurate measurement of the lead of both straight-thread and tapered ring gages. Special probes have been used for measurement of very small thread ring gages. The instrument has also been used to measure the lead of thread plug gages up to 3 in. in diameter. In general, the repeatability of the readings has been found to be as good as the precision with which the scale can be read, that is, between 0.00001 and 0.00002 in.

[1] A measuring device manufactured by Hilger and Watts Ltd. (London) is used.

NBS ELECTRO-OPTICAL IMAGE PROCESSING SYSTEM

The National Bureau of Standards has recently constructed an experimental optico-electronic system that will facilitate the study of visual perception and recognition of patterns and also promises to have a number of useful engineering applications. The device was developed by H. M. Joseph of the Bureau's electronic instrumentation laboratory in consultation with Dr. L. S. G. Kovasznay, Johns Hopkins University, who suggested the basic design and has continued to act as consultant on the project. This device can clarify blurred images or produce outline pictures, line drawings, from half tone photographs. Other possible applications include the automatic recognition of patterns such as fingerprints or biological specimens. The project was conducted as part of the NBS basic instrumentation program which is sponsored by the Office of Naval Research, the Office of Air Research, and the Atomic Energy Commission.

In the study of visual processes it is necessary to have mechanisms available which can manipulate or process images in any manner, so that the postulated mechanisms of the human brain can be approximated. For example, in the study of the recollection of patterns such a device should be able to modify a two-dimensional pattern in various predetermined ways analogous to assumed or observed processes of actual visual memory. To some extent "trick" photographic methods can be used for these purposes, but they lack the inherent rapidity, sensitivity, and flexibility which the Bureau's optico-electronic system provides.

The system operates on patterns in the form of photographic transparencies placed in an optical assembly between a cathode-ray tube with a moving spot and a photo-multiplier tube. The signals resulting rom scanning the picture are amplified and fed back to the intensity control of the cathode-ray tube and, thus, produce a picture on its screen.

The negative feedback obtained in this way improves the tonal rendition of the picture. The same signals are also applied to another amplifier and the resulting signal is used to control a monitor cathode-ray tube which reproduces the same picture. Altering the monitor for the image processing is accomplished by introducing modifying circuits between the phototube and the monitor. This freedom to modify the picture on the monitor is the essence of the system.

Any scanning pattern may be used, but it greatly simplifies the electronic circuits if the scanning velocity is the same in two orthogonal directions. The NBS system achieves this type of scan by applying triangular waves to the horizontal deflection of the scanner and a slightly different frequency wave of the same type to the vertical deflection. The result is a Lissajous figure of rectangular shape that changes its proportions with the instantaneous phase between the two waves. The same waves are applied to the monitor for identical scanning.

This image processing system has so far been employed in the study of the enhancement of contours in photographs and to the production of outline pictures from half tone photographs. The process of contour enhancement is essentially that of increasing the abruptness of tone transition at contour lines. A similar phenomenon occurs in the "brightness contrast" in human vision. If a dark area is adjacent to a light area, the dark appears darker and the light appears lighter close to the boundary. For example, the apparent gradation of tone across an individual bar in a stepped gray scale is an illusion, which can easily be verified by covering all but one bar.

The sharpening of tone transitions for contour enhancement is accomplished by modifying the waveform of the signal applied to the monitor oscilloscope. This is done by electrically adding an enhancing signal (the negative of the second derivative) derived from the original signal. Mathematical analysis of the process indicates that a first approximation to a correctly focussed picture is obtained when this process is applied to an incorrectly focussed picture. Used in this way, the system is a two-dimensional (visual) analogue of a high frequency compensated audio system.

The formation of outline pictures from photographic negatives is accomplished by using different modifying signals. If the signals from a differentiating network are rectified, a positive pulse is obtained as the light spot passes over a region of sharp tone gradient. The application of such signals to the intensity control of the monitor results in pictures that show only the contour lines, like line drawings.

This outlining process may be used for automatic production of sketch maps from terrain photographs or the display of contours on X-ray pictures or coronagraphs. In picture transmission where line drawings are acceptable, economies in bandwidth are possible through reduction in the information that must be transmitted. Also, contrast enhancement may be used in television to reduce the effects of low transmission bandwidth. The suggestion has been made that the system would be useful in the field of analogue computers as an aid to the solution of some types of differential equations.

The convenience and rapidity with which the position of the picture on the tube screen can be manipulated, as well as the possibility of scale changes in any direction, may permit automatic recognition of patterns by auto-correlation techniques. The use of outlining may facilitate this process. Among the patterns which might be recognized or compared in this way are military targets, fingerprints, printed or written matter, pathological blood cells, or various types of crystals.

NOTE: For further technical details, see L. S. G. Kovasznay and H. M. Joseph, "Processing of Two-Dimensional Patterns by Scanning Techniques," *Science*, November 21, 1953.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

May 19, 1954

The Stated Monthly Meeting of The Franklin Institute was held on Wednesday, May 19, 1954, in the Lecture Hall. The meeting was called to order at 8:15 P.M. by the President, S. Wyman Rolph. In the absence of the Secretary the Assistant Secretary, H. V. Bail, kept the minutes of the meeting. There were approximately 210 members and guests in attendance.

The President announced that the minutes of the March Stated Meeting had been printed in the April JOURNAL and those of the April Stated Meeting in the May JOURNAL and asked if there were any corrections, additions or changes in such minutes as printed. None being offered, he declared them duly approved.

The President presented to the meeting the Annual Report of the Board of Managers for the year 1953. He summarized the report for the benefit of the members and announced that it would be printed in full in the July issue of the JOURNAL.

The Secretary reported the following elections to membership for the month of April:

Active	28
Associate	22
Student	27

and announced that the total membership, including individuals with Library privileges under Company membership, amounted to 6,777 as of April 30, 1954.

The President then introduced the speaker of the evening, Dr. Froelich Rainey, Director of the University Museum at the University of Pennsylvania and formerly Professor of Anthropology at the University of Alaska, who spoke on "Radioactive Carbon and Archaeology." Dr. Rainey, who has been a member of the committee of scientists and archaeologists who have been studying the use of radioactive carbon 14 in determining the age of ancient organic materials, explained the theory and method of carbon 14 analysis and its extraordinary impact on the science of archaeology. He outlined some of the results which have already appeared from its use relative to dates in pre-history, and described the effect of these results upon the science of archaeology in general. Dr. Rainey then showed some color slides of a carbon 14 laboratory now in operation as well as some slides of some of the University Museum's current excavations.

After his formal remarks, which were thoroughly enjoyed by all those in attendance, Dr. Rainey answered questions from the audience.

The meeting adjourned at 9:45 P.M. with the President expressing the thanks of The Institute and all those in attendance to Dr. Rainey.

H. V. BAIL
Assistant Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, May 12, 1954.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 12, 1954.

PROFESSOR W. S. PARDOE in the Chair.

The following reports were presented for final action:

No. 3238: Work of C. N. Hickman.

This report recommended the award of a John Price Wetherill Medal to Clarence Nichols Hickman, of Jackson Heights, New York, "In consideration of his inventions in rocketry, in telephony, in sound-recording, in archery and in other fields."

No. 3277: Brown Medal.

This report recommended the award of a Frank P. Brown Medal, each, to Hans J. Spanner, of Zurich, Switzerland, and Edmund Germer, of Irvington, New Jersey, "In consideration of their outstanding contribution by the development of the fundamental concept of the fluorescent electric lamp, with the accompanying benefits of high efficiency, low heat output, and wide color selection."

No. 3278: Brown Medal.

This report recommended the award of a Frank P. Brown Medal to Humbolt W. Leverenz, of Princeton, New Jersey, "In consideration of his outstanding contribution to the development of the fluorescent lamp, particularly by the invention of efficient, reliable, long-lasting and versatile phosphors."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, MAY 19, 1954

ACTIVE FAMILY

Henry Corneau Beck
F. J. Colosey
J. H. W. Conklin
Lyle H. Corlette
Francis Xavier Ramil, M.D.
Harry Solomon
Meyer Solowey
William H. Warrick, Jr., M.D.

ACTIVE

Julius Abrahams
James E. Banta
James A. Croce
Perry J. Goldman
Thodore Heske
William E. Holloway, Jr.
Murray J. Makransky
Walter A. Munns
John J. O'Brien, Jr.
Seybert C. Rubin
Whitney A. Sanders
Jack V. Scott
William B. Smith
George Y. Thomson
C. B. Wentworth
John J. Winkler
Harry E. Zoll

ACTIVE NON-RESIDENT

Donald J. Hughes
Ambrose M. Keyser

NECROLOGY

Harry Berman '44
William Curtis Chubb '44
H. E. Corl '42
Edward C. Dearden '49
Pierre S. Du Pont '90
Howard George '44
Mrs. George N. Highley '36
Walter T. Karcher '52
Elmer E. Keiser, M.D. '44
Walter T. Lee '92
Mrs. Seymour DeW. Ludlum '43
Ellen Morris '36
James M. O'Neill '46
C. Frederick Rau '36
J. M. Taylor, Esq. '44
Scott P. Verrei, M.D. '49
Frank Wenner '49
F. W. Wilkening '42

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays, and Fridays; 2 P. M. until 10 P. M. on Wednesdays; 9 A. M. until 12 noon on Saturdays.

RECENT ADDITIONS

AGRICULTURE

ROE, HARRY BURGESS AND AYRES, QUINCY CLAUDE. Engineering for Agricultural Drainage. 1954.

ASTRONOMY

SMART, W. M. Celestial Mechanics. 1953.

BIBLIOGRAPHY

COZZI, FLORENCE H. Chromatography. 1953.

BIOLOGICAL CHEMISTRY

BURK, DEAN, ED. Cell Chemistry. 1953.
JOHNS, HAROLD ELFORD. The Physics of Radiation Therapy. 1953.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Catalysis. Vol. 1. 1954.
GETCHELL, ROBERT WARD. Organic Chemistry. 1954.
GOMER, ROBERT AND SMITH, CYRIL STANLEY. Structure and Properties of Solid Surfaces. 1953.
HILLER, ALMA. Practical Clinical Chemistry. 1953.
JONES, E. B. Instrument Technology. Vol. 1. 1953.
JORDAN, T. EARL. Vapor Pressure of Organic Compounds. 1954.
MASSACHUSETTS INSTITUTE OF TECHNOLOGY. MEMBERS OF THE STAFF OF THE DEPT. OF ELECTRICAL ENGINEERING. Applied Electronics. Ed. 2. 1954.
PESEZ, M. AND POIRER, P. Méthodes et Réactions de l'Analyse Organique. Vol. 1. 1952.
UNDERKOFLER, LELAND A. AND HICKLEY, RICHARD J. Industrial Fermentations. Vol. 1. 1954.

DICTIONARIES

JAEGER, EDMUND C. A Source-Book of Medical Terms. 1953.
PATTERSON, AUSTIN M. French-English Dictionary for Chemists. Ed. 2. 1954.

ELECTRICITY AND ELECTRICAL ENGINEERING

BELL, DAVID ARTHUR. Information Theory and its Engineering Applications. 1953.
LOEW, EDGAR ALLEN AND BERGSETH, F. R. Direct and Alternating Currents. Ed. 4. 1954.
MOLLOY, E., ED. Electric Motors and Control Gear. 1952.
RICHTER, RUDOLF. Elektrische Maschinen. Vol. 1–2. 1951–1953.
ROGERS, WALTER E. Introduction to Electric Fields. 1954.
STURLEY, K. R. Radio Receiver Design. Pt. 1. Ed. 2. 1953.
VOORHOEVE, N. A. J. Low Frequency Amplification. 1953.
WALL, T. F. Three-Phase Motors, Theory and Operation. 1952.

ENGINEERING

BÉTEILLE, P. Resistance des Matériaux. Pt. 1. Ed. 2. 1952.

FOOD

TRESSLER, DONALD K. AND JOSLYN, MAYNARD A. The Chemistry and Technology of Fruit and Vegetable Juice Production. 1954.

MANUFACTURE

KYLE, P. E. The Closed Die Forging Process. 1954.

MATHEMATICS

BUCHHOLZ, HERBERT. Die Konfluente Hypergeometrische Funktion. 1953.
CHEVALLEY, CLAUDE C. The Algebraic Theory of Spinors. 1954.
HODGE, WILLIAM VALANCE DOUGLAS AND PEDOE, D. Methods of Algebraic Geometry. Vol. 1–3. 1954.
JOOS, JAKOB CHRISTOF GEORG AND KALUZA, THEODOR. Höhere Mathematik. Ed. 7. 1954.
NEWSOM, CARROLE V. AND EVES, HOWARD. Introduction to College Mathematics. Ed. 2. 1954.
POINCARÉ, HENRI. Oeuvres. Vol. 1–7. 1950–1953.
REES, PAUL K. AND SPARKS, FRED W. College Algebra. Ed. 3. 1954.
U. S. NATIONAL BUREAU OF STANDARDS. NATIONAL APPLIED MATHEMATICS LABORATORIES. COMPUTATION LABORATORY. Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments. Ed. 3. 1953.
WALLOT, JULIUS. Grössengleichungen Cinheiten und Dimensionen. 1953.

MECHANICAL ENGINEERING

JÜRGENSMEYER, WILHELM. Gestaltung von Wälzlagerungen. Ed. 7. 1953.
OSBORNE, W. C. AND TURNER, C. G., ED. Woods Practical Guide to Fan Engineering. 1952.

MEDICINE

UNDERWOOD, E. ASHWORTH, ED. Science, Medicine and History. Vol. 1–2. 1953.

METALLURGY

AITCHISON, LESLIE AND PUMPHREY, WILLIAM I. Engineering Steels. 1953.

PHYSICS

BROGLIE, PRINCE LOUIS VICTOR DE. Eléments de Théorie des Quanta et de Mécanique Ondulatoire. 1953.
DEBYE, PETER JOSEPH WILHELM. Collected Papers. 1954.
RAMSAUER, CARL. Grundversuche der Physik in Historischer Darstellung. Vol. 1. 1953.
RICHARDS, C. E. AND LYNCH, A. C. Soft Magnetic Materials for Telecommunications. 1953.
SANTEN, G. W. VAN. Introduction to a Study of Mechanical Vibration. 1953.
SWAINGER, KEITH. Analysis of Deformation. Vol. 1. 1954.
VENNARD, JOHN K. Elementary Fluid Mechanics. Ed. 3. 1954.
WANG, CHI-TEH. Applied Elasticity. 1953.

WOOD

VAROSSIEAU, W. W. Forest Products Research and Industries in the United States. 1954.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

SPRATT, H. PHILIP: An Early Marine Steam Turbine Proposal.
HESSEMER, ROBERT A., JR.: Effects of Smokes on Atmospheric Ionization.
KU, Y. H. AND AHMAD KAMAL: A New Homopolar Motor.
FRANK, WALLACE E. AND ROBERT J. GIBSON: A New Pressure Sensing Instrument.
FLETCHER, S. W.: Philadelphia Leaders in Agriculture.
MOON, PARRY AND DOMINA EBERLE SPENCER: Calculation of Camera Exposure.
LINHART, J. G.: The Radiation Green's Function.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

SUGAR DISCRIMINATION BY EXPERIMENTAL RATS

BY

M. H. ROSS

Levulose is a naturally occurring physiological substance, and it was surprising, therefore, to find that some of the rats which had been offered a levulose-containing diet preferred to starve rather than eat food containing this sugar.

During the course of a study of the effects on certain metabolic processes of different amounts and proportions of dietary constituents, it was considered of interest to determine the effects of different sugars in the diet. Two diets were prepared, each containing exactly the same proportions of all the ingredients, except that one diet contained 69 per cent dextrose and the other 69 per cent levulose.

Each diet had the same physical appearance and to the observer identical taste and odor. Of the 15 adult male rats started on the levulose-containing diet, after 21 days only six survived and these showed a loss of 35 per cent of their body weight. Of the 15 rats started on the dextrose-containing diet, all were living after 21 days and showed an increase of 25 per cent in weight.

The livers of the rats which had been maintained on the levulose-containing diet showed a three-fold increase in alkaline phosphatase activity.

Since all the other conditions were maintained exactly the same, the results obtained seem to be attributable to the discrimination against the levulose.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

A 98-POINT THERMOCOUPLE DIGITAL RECORDING SYSTEM

BY

I. GLASSMAN

In the testing of jet engines, a great number of temperature measurements must be made for a particular operating condition. The conventional method of measuring and recording the low-level voltage produced by a thermocouple is to use a self-balancing potentiometer with a pen carriage attached to the drive mechanism. This scheme is fine when one is concerned with a temperature vs. time history of one thermocouple. However, in instances where a continuous measurement is not required and there are many transducers to be sampled, then a digital recording system possesses many advantages over the so called analog or graphic recording method. For these reasons, the Westinghouse Electric Corp., Aviation Gas Turbine Division, requested that The Franklin Institute Laboratories design and construct a 98-point thermocouple digital recording system to be used in the testing of jet engines.

In the last year or two, there have become available two general types of analog to digital converters. One type converts voltage to digital data, and the other converts shaft angle to digital data. For this project it was decided to use the latter type of conversion because it was more readily available and also because the voltage to digital converter normally requires a preamplifier if the voltage data are in the millivolt range. A self-balancing potentiometer was used to convert thermocouple voltage to shaft angle.

The system functions in the following manner. The thermocouples in any number from one through 98 are connected to corresponding positions on a multibank stepping switch. The output of the stepping switch feeds the self-balancing potentiometer so that each thermocouple is sampled in sequence. The digital converter which is coupled to the potentiometer motor shaft is akin to an ordinary mechanical counter. However in this application, the numbered wheels are replaced by a set of cams which actuate a ten pole electrical switch* in each decade. To carry the analogy further, if the number 5 showed up in one decade of a mechanical counter, then the number 5 switch would close in the conversion device.

* In the actual instrument, the function of a ten pole electrical switch is performed by a 10 segment commutator and a brush for each of the decades.

The digital data are now available in the form of a switch closure in each of the decades and it is now necessary to print out a digit for each of the switch closures. This is done by using an adding machine in which the keys are operated by electrical solenoids that are connected to the corresponding switches in the digital converter. The adding machine contains three columns for recording the measurement (0–999) and two more columns for printing out the thermocouple number (1–98). The solenoids for thermocouple identification are controlled by auxiliary contacts on the stepping switch.

The process of sampling the individual thermocouples, converting to digital data, and then print-out is entirely automatic. For small differences between successive temperatures, 98 thermocouple stations can be recorded in about 90 seconds. In the event that a thermocouple circuit is open, there will be no print-out, and the stepping switch will move on to the next point after a predetermined interval. Two scales (0–325° F. and 300°–600°F.) are incorporated in the self-balancing potentiometer. The scale change is accomplished automatically, and an identifying symbol is printed out when the measurement is made on the upper scale. The complete system excluding the thermocouples has a linearity and resolution of 0.1% on each of the scales. This means that out of 600° F., the accuracy of measurement after calibration is about 0.05%.

The impetus for the digital presentation of measured data is derived of course from the rapid development of digital computers during the past decade. If the analog data are to be processed by a digital computer, then the advantage of an analog to digital conversion system is plainly obvious. However even in cases where large amounts of analog data are to be averaged or operated upon in some other fashion by a "human" digital computer, the extra initial cost of digital recording over analog recording is well worthwhile.

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, EXECUTIVE DIRECTOR

EDISON'S FIRST LAMP

As this year is the seventy-fifth anniversary of Edison's carbon filament lamp, a selection has been made from the collection of lamps made by the late James D. Lee, and now in the Museum's possession, and this has been placed on display in the Museum.

It is a curious commentary upon the last two major developments in lighting (by coal gas and electricity) that their introduction should have been received with scorn by men of literary eminence. When Murdock's gas lighting was proposed for London, Sir Walter Scott wrote to a correspondent: "A madman proposes to light the streets of London with—what do you think? Smoke!" Undeterred by this false judgment by his fellow countryman, Robert Louis Stevenson went similarly astray over the introduction of electric light. In *Virginibus Puerisque* he declared the new arc lamps to be: "Horrible, unearthly, obnoxious to the human eye; a lamp for a nightmare! Such a light as this should shine forth only on murders and public crime, or along the corridors of lunatic asylums, a horror to heighten horror. To look at it only once is to fall in love with gas."

In this atmosphere of conservatism and the fear of injuring the eyes by the use of a good light, Edison proposed to make a lamp that would make gaslight look dim. Moreover, he had the wisdom to perceive that the durability and brightness of the filament was of less importance than having a system whereby each lamp was an independent unit. Such a system, he was solemnly informed by some reputable scientists, was a physical impossibility. If one arc lamp went out all the others on the same circuit were extinguished. The same thing would happen with incandescent filament lamps. Edison's genius is apparent in his achievement of the impossible feat of "dividing" electricity, rather than in his perfection of the carbon filament.

Edison was not the first in the field to make an electric lamp with a carbon filament. He began his investigation of the subject by confirming the successes and failures of his predecessors who had experimented with platinum and carbon filaments. Sir Joseph W. Swan in England had come nearest to making a practical lamp. One of his lamps is shown in the Museum exhibit.

Let us state Edison's technical problem in simple language. Carbonizing a material means heating it away from the presence of air (for combination with the oxygen in the air at too high a temperature causes combustion) so that all volatile matter is driven off and nothing remains

but a carbon residue. In the case of an electric lamp the incandescence would be the result of a current moving against high resistance in an airless space. If a metal is used, air or no air, the filament would melt at a given temperature. Carbon has the virtue of being inexpensive. To be substantial it must be solid and thick. But thickness gives it low resistance and a large amount of current must be used to bring it to incandescence. A large amount of current means that heavy wire must be used, and thick copper wire costs a great deal of money. Obviously, to make an incandescent lamp system commercially profitable a thin copper wire would have to be used, which meant the carbon filament must be very thin indeed.

Like Swan, Edison succeeded in carbonizing a piece of sewing thread and getting it inside a glass bulb, but the thread still contained enough air to make it short lived. Then began the search for a better material to carbonize. It is said Edison experimented with 6000 vegetable fibers before he hit upon a strip of bamboo. With this he made the first practical lamp, and for the first nine years all the commercial lamps had filaments of carbonized bamboo.

The subsequent development of the filament is shown in the selection of lamps on display, but visitors are advised to note the changes made in the sockets as well as in the filament. Other interesting features associated with the early electric installations are: the first type of meter employed to measure the amount of current consumed, and the three-wire dynamo used in the first public lighting system, which was installed at Sunbury, Pa., in 1883.

NOTES FROM THE BARTOL RESEARCH FOUNDATION

W. F. G. SWANN, DIRECTOR

ACCELERATION TO COSMIC RAY ENERGIES BY BETATRON ACTION *

BY

W. F. G. SWANN

In 1933, W. F. G. Swann suggested a mechanism according to which charged particles could acquire cosmic ray energies by what is now called "betatron action" in the stars. The original paper was based on rather abstract dynamical considerations, and while the validity of the conclusions is generally accepted, some have not sensed the complete implications of the theory. For this reason, a simplified development applicable to the special problem involved has been worked out verifying the original conclusions. It is also shown that difficulties raised in respect of the theory and founded upon relativistic considerations on the one hand and the high electrical conductivity of space on the other hand are without fundamental foundation.

Extension of the Acceleration Mechanism to Galaxies. It is shown that the mechanism can be extended to changing magnetic fields in galaxies and, under what would appear to be permissible assumptions as to relevant magnitudes assumed, extremely large cosmic ray energies can result.

Limitation of Interstellar Space Current Density. It is pointed out that, however great the conductivity of interstellar space may be, there is a maximum current density which cannot be exceeded for a given ionic density. This current density is nec, where n is the number of charged particles per cc. In the case where $n = 1$, the current density cannot exceed about 5×10^{-9} ampere per square centimeter. Various aspects of these considerations have been developed.

* Assisted in part by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

ABNORMALITIES IN LEAD ABSORPTION CURVES *

BY

W. F. G. SWANN

Various investigators have observed a dip in the intensity versus lead thickness curve at some value of the thickness. It turns out in a fairly obvious manner that such a dip can be related to the spectral distribution curve for momentum. In connection with the interpretation of certain data obtained by Professor P. S. Gill in India, in collab-

oration with the Bartol Research Foundation, W. F. G. Swann has given a discussion of the foregoing matter, particularly in its relation to variation of spectral distribution curve with altitude. At high altitudes the spectral distribution curve is narrow. At low altitudes it is broad, with the maximum at a larger value of the momentum.

Calculations resulting from the considerations cited have been made on the basis of existing data for the spectral distribution curve at two altitudes, 6.7 km and sea level. They serve to exhibit the fact that a dip in the absorption curve is to be expected and that it is very much more rapid at high altitudes than at sea level.

* Assisted in part by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

PHOSPHORESCENCE OF AND ENERGY STORAGE IN SOME ULTRAVIOLET EMITTING PHOSPHORS *

BY

C. E. MANDEVILLE AND H. O. ALBRECHT

Measurements of the phosphorescent decay of UV excited NaCl-Tl shows its UV and visible emission bands to decay by distinctly differing laws, whereas the bands of KCl-Tl under α-particle and UV excitation appear to have approximately the same mode of decay. The "light storage" properties of NaCl-Ag have been investigated as a function of silver concentration. The photostimulated emission of X-ray excited thick crystals of NaCl-Ag has a maximum at a silver concentration of a few tenths of a per cent by weight and shows a marked decline at concentrations as great as one per cent by weight. The energy storage properties of X-rayed beryllium oxide have been observed. By drawing short photostimulated bursts of UR from the phosphor at least once every 48 hours over a period of days, it has been concluded that BeO retains in deep traps stored electrons for a greater time (thermal escape is less probable) than does NaCl-Ag of the most favorable silver concentration. The absolute emission of BeO under both excitation and photostimulation is less than that of NaCl-Ag, but the BeO is powdered and thus has poor transmission properties.

* Assisted by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

THE INNER BREMSSTRAHLUNG IN A^{37} AND Ge^{71} *

BY

BABULAL SARAF **

The experimentally observed spectrum (by scintillation spectrometer) of the inner bremsstrahlung from A^{37} has been compared with the theoretical spectrum, after correcting for the detection efficiency, ratio of full energy peak to Compton distribution, and I-X-ray escape-peak.

The comparison shows good agreement from approximately 300 Kev to the end-point of the spectrum, but at low energies the emission seems to be larger than the theoretical predictions. The absolute source-strength of A^{37} has been measured by proportional counter, and it has been found that the emission probability for the higher energy part of the spectrum is in agreement with the theory within the probable error of about 30 per cent. The spectrum of Ge^{71} has been studied down to 20 Kev. It has been found that the emission below $\sim$ 100 Kev is much larger than the theoretical value and exceeds considerably the probable errors. The absolute source-strength has been measured by counting a known fraction of X-rays in a scintillation spectrometer. The emission probability above 100 Kev is here also found to be in agreement with the theory within the probable error of about 30 per cent.

* Assisted by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

** Research Fellow, Bartol; on leave of absence from Agra College, Agra, India.

RADIATIONS FROM Cd^{115} (43-DAYS)*

BY

JAGDISH VARMA **

The disintegration scheme of Cd^{115} has been studied by several groups of investigators with varying results. The data of the present measurements show that three gamma rays are emitted in de-excitation of levels in In^{115} at 0.95, 1.28, and 1.43 Mev. The quantum energies are 1.28, 0.95, and 0.485 Mev, the latter two in cascade. Beta-ray spectra terminate at all levels of In^{115}. The 150 Kev transition between the two higher excited states was not detected in the single counting rate or by coincidence methods. These results are in agreement with those of Engelkemeir.[1]

* Assisted by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

** Research Fellow, Bartol; permanent address Morena (M.B.) India.

[1] D. W. Engelkemeir, ANL-4717 (April, 1952).

ELECTROLYTIC DISINTEGRATION OF THORIUM OXIDE *

BY

W. E. DANFORTH

Studies of the disintegrative processes at work when high current densities are demanded of thorium oxide cathodes lead the investigator into the field of electrochemistry. The domain of electron tube engineering becomes involved by such terms as "decomposition potential" and "transport number."

Systematic observations have been made of the effects of passage of current through sintered bodies of thorium oxide at temperatures in the

operating range. One finds that electrolytic effects give rise to a transformation from semi-conductor behavior to semi-metallic behavior, a process which tends to be self-limiting as the electronic conduction increases.

Ionic effects never disappear completely, however. Recent experiments here have indicated that, for currents in excess of a critical value, electrolytic phenomena appear even in highly metallized specimens. The existence of a critical current value is presumed to correspond to a critical decomposition potential difference across individual grains of the material. Work is now in progress toward the setting up of an appropriate cell for the study of this phenomenon. Studies of the diffusion of thorium metal into thorium oxide single crystals form a part of this program.

* Assisted by the U. S. Navy Bureau of Ships.

FORWARD/BACKWARD RATIO OF SECONDARY EMISSION PRODUCED BY 1.3 MEV ELECTRONS *

BY

M. A. POMERANTZ, J. F. MARSHALL AND R. A. SHATAS

A new method for measuring the secondary-electron emission from thin targets bombarded by energetic electrons permits the independent determination of the yield from both faces of the target foil through which the primary electrons pass with negligible energy loss. The experimental arrangement comprises essentially a sandwich consisting of the target interposed between a pair of closely-spaced thin metallic foils, the electrostatic potentials of which can independently be defined. From various combinations of the 9 equations relating the 9 observed target currents to the corresponding potential configurations, the yields from the two faces of the specimen can be determined. Such an analysis in the case of an 11 mg./cm.^{2} Ni target provides a value for the ratio of the yield in the forward direction (i.e., same as incident beam) to that in the backward direction, $\delta_F/\delta_B = 1.02 \pm 0.05$, where the indicated uncertainty is inherent in the present method of measuring currents. Although the targets were only chemically cleaned, this result was unaffected by interchanging front and back surfaces by a rotation of the sandwich in vacuo. A sealed-off experimental tube, processed in accordance with the customary high vacuum techniques for insuring the cleanliness of the specimen surfaces, has now been completed. The primary beam from the linear accelerator is introduced through a thin window. Measurements are in progress.

* Assisted by the U. S. Office of Naval Research and by the U. S. Office of Ordnance Research.

ELECTRON BOMBARDMENT CONDUCTIVITY OF MgO SINGLE CRYSTAL *

BY

R. A. SHATAS, M. A. POMERANTZ AND J. F. MARSHALL

The electrical conductivity of a thin single crystal of MgO bombarded by 1.3 Mev electrons from the linear accelerator has been observed. The primary beam which penetrates the .010" target with negligible energy loss interacts with the electrons in the filled bands of the solid, causing some transition to the conduction band. A current therefrom is observed when a voltage is applied between evaporated metallic layers of the two crystal faces. The ratio of this current to the primary current is as high as 80. The internally-produced secondaries give rise to space charge effects which are being investigated in detail.

* Assisted by the U. S. Office of Ordnance Research.

TARGET-THICKNESS DEPENDENCE OF DELTA-RAY EMISSION PRODUCED BY 1.3 MEV ELECTRONS *

BY

J. F. MARSHALL, R. A. SHATAS AND M. A. POMERANTZ

The yield of energetic secondaries (delta-rays) emitted from thin foils bombarded by 1.3 Mev electrons has been determined as a function of target thickness for Al and Ni in the range 2 mg.cm.$^{-2}$ to 70 mg.cm.$^{-2}$, the upper limit being imposed by uncertainties introduced by the stopping of multiply-scattered primaries. In accordance with expectation, the yield increases with thickness from 0.5% to 2% in the case of Al of the above thicknesses. On the basis of the assumption that the lattice electrons can be regarded as free in the present analysis, and by invoking a range-energy relationship for treating the escape of the delta-rays, the yields have been computed. The theoretical absolute values are only slightly higher than the experimental results, and the observed data are in good agreement with the shape of the calculated yield vs. thickness curves. Furthermore, the difference in yields per mg.cm.$^{-2}$ of Al and Ni is in accord with theory, as is the similarity between Ni and Cu.

* Assisted by the U. S. Office of Ordnance Research and by the U. S. Office of Naval Research.

LARGE GENERATOR WORK *

BY

C. P. SWANN

The large Bartol generator is now operating very satisfactorily up to 5 Mev with about 5μa of analyzed protons. The improvement is due to the introduction of a special electrode between the ion source focus-

sing electrodes and the accelerating tube proper. The purpose of this electrode was to overcome strong defocussing which was causing the beam to strike the tube baffles and thus upset the voltage distribution. Higher voltages will be attempted soon.

* Assisted by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

INELASTIC SCATTERING OF NEUTRONS BY Ba^{137} AND Hg^{199} *

BY

C. P. SWANN AND F. R. METZGER

The principal project undertaken with the Van de Graaff generator during the past few months has been the study of inelastic scattering of neutrons through the production of the 527 Kev isomeric level of Hg^{199} and of the 661 Kev isomeric level of Ba^{137}. Neutrons were produced by bombarding lithium targets between 30 and 50 Kev thick as determined at the $Li^7(p, n)Be^7$ threshold.

The cross-section for the production of the 13/2+ level of Hg^{199} shows breaks at 610 ± 10 Kev, 980 ± 15 Kev, 1275 ± 15 Kev, and 1850 ± 25 Kev mean neutron energies. Below 610 Kev the observed activity decreases with increasing neutron energy and is attributed mainly to fast neutron capture by Hg^{198}. These different breaks indicate levels which decay to the metastable level.

The 11/2− level of Ba^{137} has been excited with a mean neutron energy of 660 ± 10 Kev, indicating direct excitation of the metastable level. Breaks occur at 1070 ± 15 Kev, 1780 ± 20 Kev, and 2370 ± 25 Kev mean neutron energies. These correspond to levels decaying to the metastable level. Indications are that at about 2200 Kev mean neutron energy there is a level decaying to the ground state more than to the metastable level causing the excitation curve to level off abruptly. Below 660 Kev the observed activity is attributed mainly to fast neutron capture in Ba^{137}.

The absolute cross-sections have been determined by making a run on the 409 Kev isomeric level of Au^{197} and comparing the results with those of Martin, Diven and Taschek.[1] The neutron yield curve of Bair, Willard, Snyder, Hahn, Kington and Green[2] was used for the relative number of neutrons. On this basis the absolute cross-sections for Hg and Ba at 3 Mev mean neutron energy are 200 mb and 600 mb, respectively.

* Supported in part by the U. S. Atomic Energy Commission and in part by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] H. C. Martin, B. C. Diven and R. F. Taschek, *Phys. Rev.*, Vol. 93, p. 199 (1954).

[2] J. K. Bair, H. B. Willard, C. W. Snyder, T. M. Hahn, J. D. Kington and F. P. Green, *Phys. Rev.*, Vol. 85, p. 946 (1952).

BOOK REVIEWS

MAGNETIC COOLING, by C. G. B. Garrett. 110 pages, diagrams, 14 × 22 cm. Harvard Monographs in Applied Science No. 4. Cambridge, Harvard University Press; New York, John Wiley & Sons, Inc.; 1954. Price, $4.50.

Adiabatic demagnetization has been, for about twenty years, an intriguing and, no doubt, a somewhat frustrating subject to many scientists. It is intriguing because it makes available to the experimenter temperatures from about 1°K. to about 0.001°K. The potential data in this range can answer many of the questions in a large variety of fields in physics. It can be frustrating because at temperatures below 1°K. experimentation is probably the most difficult to be successfully carried out. At these very low temperatures thermal conductivity is so minute that times for establishing thermal equilibrium can become prohibitive, and heat leaks of the order of 10 ergo/min. may render an experiment worthless. This amount of power is so tiny that if it were continuously available, generations upon generations of housewives would require some four million years to boil away one quart of water.

In 1940 Casimir's now classic book "Magnetism and Very Low Temperatures" was published. In the following decade a great deal of research has been done at temperatures below 1°K. and very little has been written as a detailed survey of this work. Dr. Garrett's monograph goes a long way toward satisfying the need for such a survey.

Especially interesting are the chapters on experimental techniques, thermodynamics and paramagnetism at low temperatures. The author carefully describes the procedure in setting up a demagnetization experiment. Outlined are the method of isolating the paramagnetic salt from the liquid helium bath, the type of magnet necessary to produce the appropriate magnetic fields, and the problem of reducing heat leaks. Much space is devoted to thermodynamics, specific heat, and the sometimes perplexing problem of determining temperature, and the quantum statistical theory of paramagnetism. This reviewer felt that more space could have been devoted to describing those experiments not directly concerned with the paramagnetic salts themselves; however what is lacking in text is provided in an extensive list of references.

Dr. Garrett's book provides an accurate, clearly written description of the experimental problems arising at temperatures below 1°K., of the thermodynamic theory of adiabatic demagnetization, and of many of the experiments that have been done at these temperatures. In addition there is included an up-to-date, comprehensive list of publications. This reviewer feels that this book is both interesting and informative and well worth reading.

J. GITTLEMAN

COLLECTED PAPERS, by Stephen P. Timoshenko. 642 pages, portrait, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $15.00.

The collection of Timoshenko's papers in a bound volume is of outstanding value for permanent engineering literature. The biographical sketch of this eminent engineer, written in a delightful way by D. H. Young, gives that personal charm and atmosphere in which Timoshenko accomplished his work.

While Timoshenko's work covers a wide range in mechanics, his outstanding contributions have been in the field of elasticity, both in basic theory and particularly in his masterly analysis of complicated problems to useful and practical application. This approach has been effected through the expansion in series of orthogonal functions as well as in the development of special trigonometric series meeting particular boundary conditions. His unusual mathematical and physical insight is shown in the various papers in this volume by the characterizing of a structure with the choice of suitable parameters. The influence of Lord Rayleigh's work in this type of generalization is to be particularly noted. It might be said

that one phase of Timoshenko's work is the extension of Rayleigh's theory of sound to the engineering analysis of elastic body vibration problems.

Along these lines might be cited the paper "Erzwungene Schwingungen Prismatischer Stübe" where the torsional oscillation of disks with heavy shafts, the vibration of cantilevers with concentrated end mass, are developed in terms of normal coordinates and are utilized in Lagrange's equations.

Another earlier paper is "Sur la Stabilité des Systems Elastique" published in *Annales des Ponts et Chaussée* with the application of variational methods to stability of columns, bars and trusses subjected to column action as well as several problems in thin plate stability.

An interesting paper on the effect of elastic impact between a mass and rod includes the elastic deformation between the ball and rod, using Hertz formula for the penetration. The effect of the reaction on the rod is in terms of the normal coordinates of that system. This paper is of fundamental importance in the general theory of impact.

This paper is followed by "Étude de la Flexion des Barres" using the parameters of appropriate trigonometric series in the equation of virtual work; a later paper in German, "Über die Stabilitat Versteifter Platten," covers a similar discussion on the stability of plates. These last papers are of importance in advancing our knowledge on the stability of thin walled structures.

A short but important paper "On the Correction for Shear of the Differential Equation for Transverse Vibrations of Prismatic Bars," shows the effect of shear deformation can be considerably more important than the rotational inertia correction, which latter was the only correction previously recognized.

Timoshenko's first American paper was on torsion of crankshafts, with two parts, spaced about two years apart. The first part covered the analysis of torsion in single throw shafts in the different conditions of bearing constraint. The latter covers both bending and torsion in a multi-throw crankshaft. These papers are a decided advance on the analysis of crankshafts.

In spite of a considerable expansion of interest in other fields, Timoshenko has maintained interest in structural problems. These are shown in the analysis of bridge design and rail stress problems under dynamic loadings and vibration. The paper on "Method of Analysis of Statical and Dynamical Stresses in Rail" considers a beam on an elastic foundation approximating the closely spaced tie reactions. The more complicated problem of lateral reactions on rail is also considered, together with the effect of undulations on the vibration of non-supported spring parts. The paper on "Vibration of Bridges" follows the Rayleigh procedure in using Lagrange's equations for the analysis of the effect of pulsating, moving and moving pulsating forces. It does not cover the more complicated problem of a moving mass, except for the special case of a rolling load on a flexible beam.

An important paper on "Theory of Suspension Bridges" is a masterly contribution, in which the interaction between the rather rigid truss frame and the very elastic cable is developed into basic differential equations, from which stresses in the cable and truss and deflection can be obtained. It is of interest to note that this paper won the Levy Medal at The Franklin Institute.

The author has also covered interesting subjects such as "Stress Concentration at Fatigue Failures," "The Stability of the Webs of Plastic Girders," "Stress Concentration Produced by Fillets and Holes," "The Pendulum Hardness Tester" and the "Theory of Bending, Torsion and Buckling of Thin Walled Members of Open Cross Section."

The collected papers of Timoshenko will prove of stimulating interest to all engineers.

RUPEN EKSERGIAN

CHROMIUM—METALLURGY OF THE RARER METALS, No. 1, by A. H. Sully. 272 pages, diagrams, 15 × 22 cm. New York, Academic Press, Inc.; London, Butterworth Scientific Publications; 1954. Price, $5.50.

The science of metallurgy is growing at such a fast rate and new experimental data are being obtained in detail for so many metals and alloys that the metallurgist is being forced more and more into highly specialized aspects of his field. It would be

impossible to prepare a book on all of the alloys; however a comprehensive, modern text on the individual metals that go into these alloys is possible and would be of great importance to the metallurgist, physicist, chemist and engineer.

This book is the first of a series designed to cover the rare, or more unusual metals such as chromium, zirconium, titanium, molybdenum, etc. Although there are modern texts on some of these metals published in the United States, there is no directed effort to cover the rarer metals systematically, nor is there at present an up-to-date book on chromium.

The book is opened with a discussion on the history, occurrence, and production of chromium, followed by a section on the physical properties of pure chromium. This information is particularly valuable, in that it gathers in one place the latest values obtained for many of the physical properties of high purity chromium. It has been well established that small amounts of impurities affect the properties considerably, so that these properties are constantly in need of revision as we obtain higher purity material. For example, the values reported for the melting point of chromium vary over the temperature range from 1513°C. to 1920°C. presumably due to the effect of small amounts of impurities.

A section is devoted to the melting and casting, powder metallurgy, and workability of chromium. Chromium is a difficult metal to handle in regard to these processes and it is only with modern techniques that they have been successfully accomplished. These techniques are described in detail and the various types of apparatus for melting and casting are illustrated schematically. In connection with workability, the author discusses the ductile to brittle transition that takes place in chromium as the temperature of the mechanical test is lowered. Dr. Sully has done considerable work in this field and compares his data on chromium with some of the other body centered cubic metals. The body centered cubic metals in general exhibit this ductile to brittle transition, whereas most of the face centered cubic metals do not. This important behavior has not been fully explained and requires considerable more work on high purity metals, such as the type the author presents.

Two chapters which should be of considerable industrial interest are those concerned with electroplating and chromizing. Anyone interested in, or working with these processes will find this book an invaluable aid because of the detailed description given by the author.

The research worker is continually searching the literature for phase diagrams of alloys, thus it is extremely helpful to be able to go to a text and find in one section all of the known phase diagrams for a certain metal in combination with others. Thirty-two binary systems and several ternaries are discussed. Of course all of the phase diagrams are not available since they have not been determined; however, the author has done a thorough job in gathering and printing the latest available data on this subject.

This book is complete, well written, and contains much useful information. It is a very worthwhile addition to the metallurgist's library. R. L. SMITH

HYDROCARBONS FROM PETROLEUM, by Frederick D. Rossini, Beveridge J. Mair and Anton J. Streiff. ACS Monograph No. 121. 556 pages, diagrams, charts, 16 × 24 cm. New York, Reinhold Publishing Corp., 1953. Price, $18.50.

It seems most appropriate to begin this review with a comment on the noteworthy and successful effort which has been expended in bringing forth this book. To one who is interested, but not an expert, in the science of petroleum technology, the work of Project 6 of the American Petroleum Institute on which this book reports is most interesting, and perhaps amazing in its scope. To the expert, it would seem without question to be extremely valuable.

The API, since its inception, has sponsored a great number of projects, many of them in fundamental research. Project 6 was concerned with the fractionation, analysis, isolation, and purification of petroleum hydrocarbons. This research was done over a period of 25 years, required 300 man years of work and resulted in 141 publications. However, it should be pointed out that this

book is no mere collection or compilation of the papers and reports, as the authors might have been tempted to provide, and as has unfortunately resulted in many analogous cases. Instead of a disconnected series of publications, we find a carefully written and logical succession of chapters. Each of the latter is on a separate phase, is obviously the result of careful perusal of all project data and experience on that phase, and is written with the intention of providing an integrated and clear picture of the methods and results peculiar to that subject.

As is probably well known, the principal problem in dealing with crude petroleum is the separation and purification of its component compounds or "cuts." It is the ramifications of research designed to accomplish this, with which Project 6 and the book deals. Since practically all petroleum refining is accomplished by distillation, we find most of the chapters dealing with various aspects of this technique. The theoretical considerations are clearly explained, and experimental details, including numerous complete drawings of apparatus, given freely. Chapters are also devoted to other methods of separation. These include adsorption, crystallization and extraction.

Most of the other chapters are concerned with identification of the components, such as by freezing points, boiling points and analysis. These include the important physical properties (e.g., boiling point) of what would seem to be an exhaustive list of hydrocarbon compounds.

The remaining chapters describe the fractionation of several selected crude oils, and present a complete percentage analysis of various fractions which contain perhaps hundreds of components.

This book and the research it represents are impressive, and help to reflect and explain the success of America's petroleum industry. ALBERT L. MYERSON

DIFFERENTIAL EQUATIONS IN ENGINEERING PROBLEMS, by Mario G. Salvadori and Ralph J. Schwarz. 432 pages, 15 × 22 cm. New York, Prentice-Hall, Inc., 1954. Price, $8.65.

The author's aim is rather precisely stated in the subtitle to this volume: The Application of Differential Equations to Actual Engineering Problems. This slant is not entirely new since many texts make similar claims. However, most are a far cry from their professed motto. Not so our two Columbia University professors! The guiding thought behind their work was truly the need of engineers in their work. First, certain aspects of the run-of-the-mill courses in differential equations were simply dropped. Existence theorems and other purely mathematical niceties are conspicuous by their absence. But we do find subjects not usually treated at the Junior Year level: Laplace Transforms, Fourier Series and Higher Transcendental Functions. Second, the authors gave much thought to the needs of engineers and chose their topics accordingly, including even non-linear vibrations, low-pass, and high-pass filters.

The organization of the book is in line with most texts. An introductory chapter is followed by "setting-up-exercises" for ordinary differential equations. The next two chapters deal with homogeneous and non-homogeneous linear differential equations. Chapter V introduces Laplace transforms and initial value problems. Then come simultaneous linear differential equations and integration by series. Chapters VIII and IX treat Bessel functions, elliptic integrals, and their most important properties. The tenth chapter discusses the solution of differential equations by Fourier Series and the final chapter gives an introduction to partial differential equations.

The examples are taken from every nook and cranny of the engineers' field of activity. To mention a few, we find thermodynamics, chemical reactions, branch and mesh currents, vibrating strings, cables, transmission lines, deflection and buckling of beams, and many others. Noteworthy mathematical techniques included are the already mentioned chapter on Laplace transforms and suitable tables, operator methods, and Fourier expansions with graphs for a large number of useful periodic functions.

By actual count, 687 well graded problems are distributed over the eleven chapters; some of these problems contain as many as ten or more individual exercises. The necessary expository material is clearly written and elaborates on both physical laws

and engineering concepts. The volume follows the modern trend of "integrated" texts and will be appreciated by students of applied mathematics or engineering and teachers alike. CARL HAMMER

CELL CHEMISTRY, edited by Dean Burk. 362 pages, tables, illustrations, 18 × 26 cm. Houston, Elsevier Press, Inc., 1953. Price, $7.50.

This book is dedicated to Otto Warburg on the occasion of his 70th birthday. It consists of 36 original research articles by many renowned scientists and a brief biographical introduction by Dean Burk. These articles first appeared as a supplement of the journal, *Biochimica* et *Biophysica Acta* (Vol. 12, No. 1/2, 1953). This book is identical with the journal supplement with the exception that the bound volume has a subject index and an author index, each one page long. Two of the papers are in French, five are in German, and the rest are in English. The summary of each paper is presented in all three languages.

The investigations reported here deal with the chemistry and metabolism of carbohydrates, fats, proteins, nucleic acids, and their derivatives. Some of the biological materials which were studied are bacteria, algae, plants, animal tissues, and ascitic tumors. There is no continuity from one article to the next; they appear to be presented at random.

Warburg is the author of one article and a co-author of another, both of which are on the measurement of the quantum requirement of photosynthesis. The many references to his discoveries attest to the importance and influence of Warburg's scientific achievements. W. D. YUSHOK

INTRODUCTION TO DYNAMICS, by L. A. Pars. 501 pages, diagrams, 15 × 22 cm. New York, Cambridge University Press, 1953. Price, $6.00.

Textbooks written after giving a course of instruction for several years always bear a mark of experience different from those written preceding a course of instruction. This fact is borne out in *Introduction to Dynamics*, which is based upon a series of lectures on elementary dynamics given by the author at Cambridge University. The book, written for first year graduate students in mathematics and physics, has as its scope the study of motion in two dimensions, without Lagrange's equations. The reader is assumed to have a knowledge of elementary statics and a working knowledge of calculus.

As a general introduction to the subject of dynamics, the text is successful. The separate topics considered in the text are usually introduced by a discussion, with examples, of special cases. These lead to the general theory pertaining to a particular subject. It has evidently been the author's experience that this technique quite readily results in obtaining and keeping students' interest. Since the purpose of the book is to get the reader to understand dynamical principles, and not mechanical manipulation of mathematical techniques, vector notation is used only where the author thought that such notation would simplify or clarify a presentation. Otherwise algebraic notation is used.

The twenty-six chapters can be roughly divided into ten topics. The first three chapters introduce sufficient vector topics to satisfy the needs of the rest of the book. The relatively simple problems of particle motion along a straight line and in response to fields of force and the attendant energy relationships are considered in eleven chapters. Emphasis is placed upon problems dealing with conservative fields, with examples drawn from the topics of pendula, projectiles, central orbit fields and inverse square law fields. Only one chapter is devoted to the topic of variable mass.

Two chapters devoted to particle motion in a plane and another to the motion of a lamina introduce four chapters dealing with the forces on rigid bodies and three on the motion and momentum of systems. Since Lagrange's equations are not used, only limited examples can be used; however, there are sufficient of these to illustrate the fact that what is important is an understanding of the principles of dynamics.

Liberal use is made throughout the book of worked-out examples to introduce special topics. Numerous examples are included at the end of each chapter which are bound to stimulate the reader's thinking. Many of these have practical application, which enhances their interest. S. CHARP

THE PAPERS OF WILBUR AND ORVILLE WRIGHT, edited by Marvin W. McFarland. Vol. 1, 1899–1905; Vol. 2, 1906–1948. 1278 pages, port., diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $25.00.

The publication in 1953 of these two volumes containing most of the letters and notes of these pioneers of human flight, together with the exchange of correspondence between them and Dr. Octave Chanute, forms a most fitting climax to the celebration of the 50th year of flight. Mr. McFarland is to be congratulated on the job he has accomplished. The large amount of "foot-noting" bears testimony to the care he has taken to insure continuity, and provide understanding by the reader of what is, except for the Chanute letters, only one side of the picture.

As for the letters themselves, they are most fascinating reading for anyone interested in aviation, and give as no other method could, a clear picture of these two remarkable young men—simple, honest, unaffected, and yet dedicated to a task from which they could not be swerved.

No one can read these letters and still believe the old myth that the secret of heavier-than-air flight was just a lucky shot by a couple of untutored bicycle mechanics. In these letters we obtain an intimate picture of two born scientists, proceeding in orderly research toward their goal—Orville, the shy, quiet thinker; Wilbur, the more aggressive, the spokesman of the pair. Probably the traits which contributed most to their ultimate success were patience and persistence under adversity and failure.

To us, with aviation all around us, the tremendous difficulty the Wrights had in convincing the world and particularly our military service—or even, for that matter, their own home town of Dayton—that they were flying successfully is almost unbelievable.

All this, together with the story of the Wrights' long fight for recognition of their basic patents, is told in detail in these comprehensive volumes.

Aviation and the world sustained a great loss in Wilbur Wright's untimely death in 1912. An insight into his methods is obtained from one of his letters in which he states:

> If a man is in too big a hurry to give up an error, he is liable to give up some truth with it, and in accepting the arguments of the other man he is sure to get some error with it. Honest argument is merely a process of mutually picking the motes and beams out of each other's eyes so both can see clearly. Men become wise just as they become rich, more by what they save than by what they receive. After I get hold of a truth I hate to lose it again and I like to sift all the truth out before I give up an error. RALPH S. BARNABY

LES APPLICATIONS DE LA MÉCANIQUE ONDULATOIRE A L'ÉTUDE DE LA STRUCTURE DES MOLÉCULES, edited by Louis de Broglie. 223 pages, 16 × 24 cm. Paris, Revue d'Optique Theorique et Instrumentale, 1953. Price, 1600 fr. (paper).

This is a collection of articles on the application of wave mechanics to the study of the structure of molecules. The papers are for the most part survey papers of topics in this field. They were read at conferences held in April–May–June 1951, presided over by Louis de Broglie. In the introduction, de Broglie describes how in early quantum theory, electrons were considered as points describing trajectories about the nucleus. However, in wave mechanics, there is a wave ψ associated with each electron such that $|\psi|^2$ at each point gives the probability of the electron being at that point. In the first paper Daudel gives an outline of the theory, describing the wave equation for a molecule of N electrons and n nuclei and describing the methods of solution of the wave equation.

We list the titles and authors of the papers in this collection:

Recent progress in infrared spectrometry, by Jean Lecomte.

Determination of interatomic distances by X-ray and electron diffraction, by Jean Wyart.

Calculation of interatomic distances, by O. Chalvet and C. Vroelant.

Electronic structure of carbon and nitrogen, by Pauline Ramart-Lucas.
Theory of electronic spectra of organic molecules, by C. Sandorfy.
Electric polarization and dipole moments, by Nguyen-Quang-Trinh.
Electric moments and quantum resonance, by H. Lumbroso.
Methods of calculation of dipole moments, by A. Laforgue.
Magnetic properties and the structure of molecules, by A. Pacault, N. Lumbroso and J. Hoarau.
Is it possible to see atoms and molecules with the microscope? by P. Chanson.

This book would be useful to workers in this field since it summarizes the results up to 1951.

H. L. PLATZER

PROGRESS IN NUCLEAR PHYSICS, Vol. 3, edited by O. R. Frisch. 279 pages, diagrams, 16 × 25 cm. New York, Academic Press, Inc., 1954. Price, $9.50.

This is another in the series of extremely useful review collections. The nine articles have the excellence we have come to expect by virtue of the high caliber of the previous volumes. These volumes are essentially British productions, although one American, Martin Deutsch, is listed among the authors of this latest collection. Four of the articles are concerned with detectors, one with ion sources, one with nuclear spins, one with positrons and two with deuterons. Unlike the review articles in the American counterpart,* these articles are strictly in the field of nuclear physics.

The articles and their authors are as follows:

1. The diffusion cloud chamber, by M. Snowden
2. Energy measurements with proportional counters, by D. West
3. Oriented nuclear systems, by R. J. Blin-Stoyle, M. A. Grace and H. Halban
4. Čerenkov radiation, by J. V. Jelley
5. Annihilation of positrons, by Martin Deutsch
6. Solid conduction counters, by F. C. Champion
7. Stripping reactions, by R. Huby
8. The production of intense ion beams, by P. C. Thonemann
9. The collision of deuterons with nucleons, by H. S. W. Massey.

Editor Frisch has included subjects which are of current interest. As a detector, the continuously sensitive diffusion cloud chamber is much superior to the expansion type. Proportional counters have reached a high state of perfection despite their need for calibration. They have the advantage of good resolution at low energies. Čerenkov detectors have properties with respect to speed, efficiency and discrimination possessed by no others. The solid conduction counter, however, is still in its infancy but its study may lead to a better understanding of the solid state.

It is rather amusing to find that Čerenkov, a Russian, was not the first to observe the radiation named after him. Dr. Jelley gives an informal review of this visible radiation produced by the passage of fast charged particles through matter.

The section on oriented nuclear systems seems to be somewhat short for such a large subject. Topics discussed are production and properties of oriented nuclei and beams.

Professor Deutsch discusses positrons, their annihilation, and positronium. The latter is a hydrogen-like system of a positron and an electron.

Each section has an introduction, a theoretical development, a discussion of design principles if the section is on a research tool, a part devoted to experimental techniques or results, a part on applications, and finally a summary or conclusion. The result is that each section stands on its own feet as a short monograph.

The book is well-indexed, has excellent illustrations and plates, and is typographically easy to read. It is recommended for those interested in keeping up-to-date in nuclear physics in a time-saving manner.

LEONARD MULDAWER

HOW THE EARTH IS MADE, by Peter Hood. 64 pages, plate, illustrations, 17 × 24 cm. New York, Oxford University Press, 1954. Price, $2.50.

This is the third book to be published in the Oxford Visual Series of physical and

* *Annual Review of Nuclear Science.*

natural sciences and four of the six are by the author.

After writing an elementary astronomy book in "Observing the Heavens" there came a book on "The Atmosphere" which was slanted toward the meteorology field. Now comes this book on the earth.

"How the Earth Is Made" is a misnomer for the book implies a genesis. Actually the book relates in its entirety to the surface layers of the earth. The reviewer includes volcanism and earthquakes as surface details.

It deals with rocks, their formation, their erosion to form the soil. The motions of the earth's crust both in the long time scale to create mountains and the short time scale for earthquakes are included.

The appearance of life and the evolutionary trends of both animals and men are described. Finally there is a section on ice ages, mining and geology of Great Britain.

What appears in the book is well written for the author has a fortunate faculty of painting pictures with words. As though that was not enough, there are incorporated both illustrations and photographs which enhance the value of the book. Unfortunately in the sixty pages of this short book he has incorporated so many topics that the book has been transformed into a pot pourri of isolated topics. This reviewer is certain that the author could have expanded any one of the 28 chapters and have written the thirty pages of text on it.

In spite of this shortcoming the book is a pleasant little volume which youngsters will welcome. I. M. LEVITT

PHYSICS FOR MEDICAL STUDENTS, by J. S. Rogers. Third edition, 405 pages, illustrations, 14 × 22 cm. New York, Cambridge University Press; Melbourne, Melbourne University Press; 1954. Price, $5.50.

The avowed purpose of the text as stated by the author in his preface is to supplement those textbooks which have been written for first year university students studying for the degrees of Science and Engineering. This purpose has been accomplished. The text is a good supplement especially for pre-medical students. All the subjects considered in the text are treated from a fairly fundamental point of view, but in a most elementary way. As far as this reviewer could find, there is not one problem or exercise for the student to do. This automatically removes it from the texts which could be used as the basis of a course. The book could have been made more interesting as a supplementary text if the illustrations were more descriptive. For the most part they consist of simple line drawings. In some ways this volume seems more like a compendium than a text for general study. Practically every page contains references and quotations from workers in the particular subject being discussed.

This text is very definitely aimed at the pre-medical student. The first two chapters are purely historical. The student is taken from physics in pre-Grecian times through Pythagoras, Aristotle, Archimedes, Alhazan, Copernicus and Newton, to mention a few, in the first chapter and is brought up-to-date —that is, to the Second World War—in the second chapter. This is a good historical review and will help any science student to develop a greater interest and integration of science. Some chapters directed toward the pre-medical student are: "Hydrogen Ion Concentration" (usually studied in physical chemistry courses), "Blood Pressure and its Measurement," "The Electrocardiograph and the Electrocephalograph" (both subjects rarely found in general physics texts) and "The Therapeutic Use of X-ray and Gamma Rays."

Some sections of chapters have been marked as too difficult for the less gifted students—in particular, the section on phase-contrast microscopy. This reviewer feels that this is a bit out of line, since this section is no more difficult and far more interesting to the pre-medical student than the following section on the reflecting telescope. This poor differentiation occurs in other places, also.

In general, the author has succeeded in producing a supplementary text of value in stimulating the interest of pre-medical students in physics. This text, however, could be used only as an adjunct to a good problem course in basic physics. R. J. GIBSON

CATALYSIS, VOLUME I. FUNDAMENTAL PRINCIPLES (PART I), edited by Paul H. Emmett. 394 pages, illustrations, 16 × 23 cm. New York, Reinhold Publishing Corp., 1954. Price, $10.00.

This book initials a series of monographs which will survey the present status of the field of catalysis under various subject headings written by appropriate authorities. The project is ambitious in scope and virtually all phases of the field will be included. The present volume comprises the first half of a discussion on fundamental principles. The topics covered are: Chapter 1, Physical Adsorption, by Herman E. Ries, Jr.; Chapter 2, Measurement of the Surface Area of Solid Catalysts, by Paul H. Emmett; Chapter 3, Chemisorption, by Keith J. Laidler; Chapter 4, Kinetic Laws in Surface Catalysis, by Keith J. Laidler; Chapter 5, The Absolute Rates of Surface Reactions, by Keith J. Laidler; Chapter 6, Catalyst Preparation, by F. G. Ciapetta and C. J. Plank; and Chapter 8, Magnetism and Catalysis, by P. W. Selwood.

The first chapter is a concise review of van der Waals adsorption of gases on solids. The subject is discussed from the point of view of the adsorption isotherm beginning with definitions and tracing it down to present day ideas of multilayer adsorption and to the role of mobile layers and interactions between adsorbed molecules. A section on capillary condensation and its relation to pore structure is included.

The methods useful in the measurement of surface areas are discussed in the second chapter, the BET method being presented at length and analyzed critically. The article is sufficiently detailed to give the reader a working knowledge of the problem and is especially valuable in its impartial appraisal of the methods available for surface area measurement.

The chapter on chemisorption can be considered an expansion of Chapter I, for it compares both the kinetic and statistical treatments of the adsorption isotherm. Those features common to both types of adsorption but more relevant to chemisorption are treated fully, namely surface heterogeneity and its relation to activation energy. The important work of Roberts is reviewed and a valuable summary, in table form, of existing work on chemisorption is given. The overlapping of some of the material in Chapters I and II might have been avoided if the subjects had been treated under one heading.

The next two chapters are unquestionably a main feature of the book, and justifiably deserve the quarter of the book devoted. The one on kinetic laws applies basic principles of chemical kinetics to surface reactions, in other words demonstrates how the rate equation may be determined from experimental data taken under different conditions, and how the order, molecularity and mechanism of a reaction may be deduced from the rate equation. These ideas are illustrated by about twenty examples of work taken from the literature. This feature serves independently as an excellent survey of heterogeneous reactions of gases on solids. The treatment on the absolute rates of surface reaction might be more profitably read with a previous familiarity with the absolute reaction rate theory (and its limitations) but this is not essential. The material parallels that of the previous chapter and in fact was designed to show how the theory is useful in assigning fundamental significance to the quantities in the kinetics equation.

The chapters on promoters and related topics and on catalyst preparation are descriptive but the excellent organization and correlation of the voluminous information deserves special commendation. Clarity and completeness are achieved by superlative charts and photographs in the chapter on promoters. The chapter on catalyst preparation actually includes working formulas for fabricating common types of catalysts. The chapter on magnetism is a short review devoted chiefly to explaining very briefly the usefulness of magnetic techniques in the elucidation of catalyst structure and the relation of magnetic to catalytic properties. The material is a condensation of Selwood's own article in Volume III of "Advances in Catalysis."

This bald summary cannot do justice to the excellent organization of each topic in the book and the careful thought that went into developing each. Although the treat-

ment is general in no sense is it superficial, while mathematical obscuration is nonexistent. Speculation is present only to suggest additional avenues of investigation. To the active worker in the field a valuable contribution will be the ubiquitous use of tabulation in summarizing data and information. We hope this will remain a characteristic of the series. References are complete, and author and subject indices are included. O. D. GONZALEZ

INTRODUCTION TO ELECTRIC FIELDS, by Walter E. Rogers. 333 pages, drawings and photographs, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.50.

Although there are many excellent texts such as those by William R. Smythe, J. A. Stratton and others dealing with electric fields, Professor Walter E. Rogers at the University of Washington has succeeded in preparing a short volume which contains two interesting variations in its presentation of this subject.

The first of the innovations is the use of field models. These models are remarkably graphic and add much to the student's interpretation of the formal mathematics. Electrical engineers will recognize the field models as applications of a technique called Fluid-Flow Mapping described by Professor A. D. Moore of the University of Michigan. Professor Moore announced his methods of field mapping in the August, 1949 issue of the *Journal of Applied Physics* and has subsequently presented refinements of this method in various electrical engineering journals.

The fluid-flow field map is produced by a moving film of water made visible by dying it with slowly-dissolving, brightly-colored crystals of potassium permanganate sprinkled upon a white background upon which the water flows. One of the most valuable uses of these models is in the study of complex fields which can be studied only with great difficulty in purely mathematical terms.

A second form of illustration employed by Professor Rogers is the Rubber-Sheet Model of two-dimensional potential fields. This technique has been employed by investigators of electron beam behavior, notably Zworykin, Spangenberg, Weber, Kleynen and Pierce.

In both of these model techniques, of which the author makes such extensive use, it was the reviewer's hope that a full explanation of the analogies between the velocity of a slowly-moving incompressible fluid or the deflection of an elastic membrane and the electrostatic field might be given. Surely the demonstration of these analogies in vectorial terms, which are essentially the method of this text, would have been interesting and would have made the models more meaningful to the student. In fact, the author promised in the early part of the book to carry out such a demonstration after he had treated Laplace's and Poisson's equations, but this appears to have been overlooked as the manuscript progressed.

The second point of difference perhaps between Professor Rogers' book and other standard works is the persistent use of the vector analysis approach. The reader will realize as he uses this book that it contains within its relatively few pages all of the formal field theory which usually occupies a larger volume, and this seeming brevity is undoubtedly a result of the exclusive use of vector notation.

The appeal of this book as a teaching text should be broader than many other books on electric field theory because they are often devoted extensively to electromagnetic waves. As will be seen from the foregoing discussion of models, this book should have as much attraction for Power majors as for those students and teachers specializing in communications. C. W. HARGENS

SINGULAR INTEGRAL EQUATIONS, by N. I. Muskhelishvili. Second edition, translation from the Russian edited by J. R. M. Radok, 447 pages, 15 × 22 cm. Groningen (Holland), P. Noordhoff Ltd., 1953. Price, fl 28.50.

In recent years the theory of singular integral equations has assumed increasing importance in applications. This book presents the theory of one-dimensional (that is, where the range of integration is one-dimensional) singular equations involving Cauchy principal values.

The fundamentals of the theory of one-dimensional singular integral equations were developed by Poincaré and Hilbert, directly after the development of the classical theory of integral equations by Fredholm. This theory has advanced considerably in recent years. The present text is an attempt to present this theory in a finished form.

As the fundamental tools of this subject are Cauchy type integrals, part I (85 pages) of the book gives a survey of the properties and theorems connected with such integrals. The Hölder condition for a function defined on an arc L is considered in detail, since this concept is used throughout the book.

The author then presents a solution of the homogeneous Hilbert boundary problem and the Riemann-Hilbert boundary problem of the theory of analytic functions. With this effective background material, the central topic of the book—singular integral equations with Cauchy type kernels—is introduced. The properties of singular operators and their adjoints are explored. The general method of investigation of the singular integral equation consists in its reduction to a Fredholm equation. Several fundamental methods of reduction are considered.

The theory is applied to other classical boundary problems of the theory of analytic functions. These include the Dirichlet problem of potential, and the Hilbert boundary problem for the case of arcs or discontinuous boundary conditions. The inversion formulae of the Cauchy integral taken over a union of arcs are deduced.

Advanced engineering applications include development of the formulae for the plane static theory of elasticity. The general problems treated are the half-plane, circle, and the plane with straight cuts. The singular integro-differential equation of the theory of aircraft wings of finite span is discussed and solved.

A concluding chapter extends the theory to systems of singular integral equations with Cauchy type kernels.

The author is head of the Mathematical Institute of the University of Tiflis, where he has set up a leading theoretical research group of the USSR. The book is intended particularly for research workers in theory of elasticity, hydromechanics and other branches of mathematical physics. The book is an outstanding contribution to the theory of singular integral equations and applications of analytic function theory. The content is readily accessible to those acquainted with theory of functions of a complex variable and the theory of Fredholm integral equations. The writing is clear and rigorous throughout.

HOWARD H. BROWN

FOURTH SYMPOSIUM (INTERNATIONAL) ON COMBUSTION (COMBUSTION AND DETONATION WAVES). 926 pages, illustrations, 18 × 26 cm. Baltimore, Williams & Wilkins, 1953. Price, $7.00.

This book contains 103 papers, nine survey papers and summaries of two round-table discussions presented at the Massachusetts Institute of Technology, September 1952, on the physical aspects of combustion, particularly wave phenomena and turbulence, treated from both experimental and theoretical standpoints. The complexity of the subject of combustion is so great that even though only a part of it is covered in the Fourth Symposium, this reviewer will not attempt to make remarks about the technical content of the papers.

The survey papers, designed for younger workers and for those unable to keep abreast of recent developments in the combustion field, are believed to be the most valuable part of the book. The titles of these papers, indicating approximately the subject division of the rest of the symposium papers, and the names of the authorities who presented them are: 1. Limits of Inflammability by Sir Alfred C. Egerton; 2. The Problem of Ignition by Guenther von Elbe; 3. Methods of Measuring Burning Velocities by J. W. Linnett; 4. The Structure of Laminar Flames by Melvin Gerstein; 5. Instability Phenomena in Combustion Waves by G. H. Markstein; 6. Open Turbulent Flames by Bela Karlovitz; 7. Quenching, Flash-back, Blow-off—Theory and Experiment by Kurt Wohl; 8. Flame Stabilization by Bluff Bodies and Turbulent Flames in Ducts by J. P. Longwell; and 9. Burning in Laminar and Turbulent Fuel Jets by H. C. Hottel.

One finds the largest number (50) of papers under the subject heading "Theoretical and Experimental Studies on Laminar Combustion and Detonation Waves." The remainder of the papers are distributed uni-

formly among the other subjects mentioned above. The discussion of a paper, if any at the time of presentation, has been published unedited at the end of the paper. This is a valuable addition to any scientific publication and makes the record of the symposium complete. The round table discussions on the theory of the combustion wave and the combustion process in turbulent flames point up the success or lack of success of the potentially more powerful theoretical methods. It appears that considerable progress has been made in the field of flame turbulence.

In recent years there has been a remarkable growth in interest and in the number of research workers in the field of combustion. It is believed that a closer liaison between the research worker concerned with fundamentals of combustion and the engineer concerned with its utilization has been developing. The symposia initiated and organized by the Standing Committee on Combustion Symposia have done much to bring together these groups and to consolidate workers with diverse interests and training. Bernard Lewis states in his introduction that he believes the requirements of this group must be met sooner or later by a separate organization or society devoted to the interests of the science of combustion.

The Standing Committee is to be congratulated on the excellent results of its labors. Williams & Wilkins has also done an excellent job of composing and printing. The volume is the same size as that published following the Third Symposium. There are many reproductions of Schlieren, shadow and direct high-speed photographs of combustion phenomena, all of high quality. Although there may be some, this reviewer has found no typographical errors. The table of contents and the list of authors make it rather easy to find a particular paper or subject.

Although the attendance at these symposia is rapidly growing, there are undoubtedly many more unable to attend who appreciate having a symposium brought to them in the form of this excellent printed record. Let's hope the records of future combustion symposia will continue the high standards set in this volume. W. E. SCOTT

EXPERIMENTAL INORGANIC CHEMISTRY, A GUIDE TO LABORATORY PRACTICE, by R. E. Dodd and P. L. Robinson. 424 pages, diagrams, 16 × 23 cm. Houston, Elsevier Press, 1954. Price, $6.50.

If nothing else, this book illustrates the ingenuity of generations of laboratory workers in devising apparatus for countless experimental purposes. It is all the more impressive when it is realized that a book such as this can do no more than scratch the surface of this subject. Actually the techniques and methods discussed are just as pertinent to physical chemistry as to inorganic chemistry. Thus a chapter is devoted to vacuum technique; another to physical methods (principally measurement of physical constants). The central section (Chapters 3 and 4) constitutes the truly inorganic part of the book. These two chapters consist of a description of the properties and preparation of a selected list of inorganic compounds (about 160 in all) ranging from water to nitryl fluoride. Chapter 3 deals with "gas and volatile compounds" while Chapter 4 covers "water, alkali, acids, solvents." This, of course, excludes whole classes of inorganic compounds. Each compound, on the average, is allotted something less than a page.

Treatment of some of experimental methods is likewise exceedingly brief. Thus polarography is disposed of in two short paragraphs while X-ray, electron, and neutron diffraction are together allotted only five pages. It would seem wiser to leave such topics out altogether rather than treat them so briefly.

A footnote on each page gives the location at which the references cited on that page are to be found. The bibliography is extensive. E. R. STEPHENS

FOAMS: THEORY AND INDUSTRIAL APPLICATIONS, by J. J. Bikerman, with R. B. Booth, J. M. Perri and C. C. Currie. 347 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1953. Price, $10.00.

Foams have been used for centuries without much regard to the science of their formation, structure, and properties. With increasing industrial application of foams, a

technology was built up based partially on scientific fact but mostly on empiricism, rule of thumb, and plain guesswork. Coupled with this new technology was a wealth of tests, test data and theories on foams scattered through the literature. Many of these publications on foams were not only confusing but, in many cases, also contradictory. Dr. Bikerman, a well known specialist in the field, has chosen this book to put the disorder of foam technology into order and to try to extract some basic principles by which to explain foams.

The first section of this book deals with the theory of formation and the structure of foams, methods of measuring and explanation of foaminess, foam drainage, the mechanical, optical, and electrical properties of foams, and the fundamental concepts behind some of the industrial uses of foams. The second part of this book, comprising only about one quarter of the whole book, is devoted to a review of the most recent developments in industrial applications of foams. The chapters in this section have been prepared by specialists in the particular application fields. The subjects covered are fire fighting foams, froth flotation, and chemical antifoaming agents.

This book accomplishes, at least to some extent, its goal of improving the understanding of foams. There is, however, such an enormous amount of patents, experiments, experimental data, conclusions, and theories regarding foams that the book tends to be a series of short essays discussing and trying to explain the results of various experimenters. Probably the subject matter could not have been presented otherwise owing to the nature of the field. The chapters on industrial applications, however, are better organized but the subject matter is also much more clearly defined. Dr. Bikerman's book does satisfy the need in the field for a complete, up-to-date, critical review of foam technology. It also provides a valuable source of information on the subject to students, research men, and those involved in the use of or combating of foams. Most important though is that it represents a concerted effort to organize the science of foams and to start those concerned with foams thinking in terms of the fundamental principles rather than only prior art.

DONALD H. RUSSELL

ENGINEERING STATISTICS AND QUALITY CONTROL, by Irving W. Burr. 442 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $7.00.

This book is the outgrowth of a course in statistics for engineers and is written as a textbook for junior, senior and graduate students of engineering and physical science. Its principal topic is the application of statistical methods in industrial quality control.

A brief introductory chapter is followed by three chapters serving to introduce the reader to the concept of a frequency distribution and to its characterization by the first and second moments as well as the range of variation. The properties of the Gaussian distribution are discussed and the use of the chi-square criterion in the test for normality of an empirical distribution is described.

The subsequent three chapters deal with Shewhart control charts for measurements and the methods for setting up the appropriate center and control-limit lines. The concept of a process in "statistical control" is introduced and the indications of lack of control due to "assignable causes" are discussed at some length. An outline of the theory of control charts is given.

Chapter 8 serves to introduce the reader to the mathematical theory of probability. "Statistics and probability are so fundamentally interrelated that it is impossible to draw even a shadowy line where one leaves off and the other begins." This includes definitions of *a priori* probability and of *statistical* probability (that is, the frequency interpretation of probability, which the author chooses to call the *a posteriori* probability!), some combinatorics, addition and multiplication of probabilities, and the binomial, the hypergeometric and Poisson distributions.

The next three chapters deal with the problem of control charts for "attributes" (that is, fraction of defective pieces in a lot or number of defects in individual pieces of a lot), of the use of slanting control lines in the case when known trends are present, of the choice of an appropriate sampling technique, and of "putting control charts to work."

Chapters 12 to 15 are concerned with acceptance sampling for attributes and by

measurements under various conditions occurring in practice.

The last chapter contains a discussion of the place of statistics in industry and a "thumbnail sketch" of statistical tools other than control charts. A brief section giving some general suggestions throws an interesting sidelight on the role of management in statistical quality control.

The Appendix contains a Glossary of Symbols adopted as standard by the American Society for Quality Control, as well as fifteen tables of various functions which are required for the numerical work in this field. A subject index is given at the end. Numerous worked examples given throughout the text and problems to be solved by the reader appearing at the end of most chapters serve to illustrate the detailed mathematical work involved in the various phases of statistical quality control.

This book compares very favorably with other introductory texts on quality control: the material is presented in a systematic manner and with great didactic skill. It is particularly well suited as a text for a training course whose primary purpose it is to make the student familiar with the existing quality-control techniques.

Without wanting to detract from the value of this book, this reviewer feels moved to make the following remarks regarding the general subject of statistical methods in engineering: it is highly regrettable that, despite the remarkable advances in the mathematical theory of probability during the past thirty-or-so years, this subject is not a part of the regular engineering curricula. As the author puts it (p. 393): ". . . statistical reasoning requires something of a revolution in thinking." This type of reasoning is now familiar to most physicists and to some communication engineers and economists, but it is obvious to any one familiar with the concept of a random variable, that there is almost no field of human endeavor in which this type of reasoning could not be profitably used. The scope of probabilistic methods is much wider than may be inferred from the author's "thumbnail sketch" of other statistical tools (pp. 394 to 396). The universities should be the centers for the dissemination of probabilistic concepts and methods throughout industry and other sectors of the social organism. Yet, the great potentialities of "statistical thinking" are, at present, not generally recognized. In the light of these remarks, this book appears as a product of an era which is drawing to a close, rather than as a harbinger of the future in "engineering statistics."

OTTO R. SPIES

THE SCREEN PROJECTION OF CHEMICAL EXPERIMENTS, by E. J. Hartung. 291 pages, plates, 15 × 22 cm. Melbourne, Melbourne University Press; New York, Cambridge University Press; 1953. Price, $4.75.

This well written volume, intended as a guide for teachers of chemistry, could also be useful in industrial laboratories. Unless one has had occasion to apply the principles of elementary optics frequently, the detailed information contained in this book could save many tedious hours of search.

The first two chapters describe in detail the fundamentals of projection optics. Amply and clearly diagrammed, these chapters show the application of these principles to the various types of optical projection. These types include Shadow, Diascopic, and Episcopic projection, the requirements to obtain parallel light, and the several types of microprojection. Details of the theoretical and mechanical requisites for projection in polarized light are presented in readable form.

Two chapters are devoted to mechanical details. Such components of an optical assembly as light sources and circuits, electrical equipment for conversion of AC to DC for use with these light sources, lenses and mirrors, optical benches, and the other paraphernalia that is necessary in a complete optical assembly are discussed in a practical fashion.

The rest of the book describes two hundred and fifty experiments where the application of this type of demonstration can be used advantageously. The subjects include demonstrations of the properties of gases, liquids, and solids. Other experiments are presented to illustrate the basic principles of physical chemistry, analytical, electro, surface and photochemistry. All the experiments are indexed for each reference.

The reviewer considers this short volume of two hundred and ninety-one pages a valuable addition to any chemical laboratory library.

SAMUEL N. MUCHNICK

BOOK NOTES

ENGINEERING CONTRACTS AND SPECIFICATIONS, by Robert W. Abbett. Third edition, 429 pages, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1954. Price, $6.00.

All legal aspects of construction work are clearly set forth in this third, revised, edition of a book which has proved its value to contractors and practicing engineers. New material has been included on engineering and construction law, on cost plus fixed fee contracts, on contracts for architectural and engineering services, and on specification writing. The book is recommended as a reference and as a basic, usable legal guide to the administration of construction work.

HISTORY OF AMERICAN INDUSTRIAL SCIENCE, by Courtney Robert Hall. 453 pages, 15 × 22 cm. New York, Library Publishers, 1954. Price, $4.95.

Much of the material for this history of industrial science was furnished by sixty of America's leading corporations. This array of statistics has been interpreted for the lay reader as well as the industrialist, by the author, a professor of American history for 25 years. Dr. Hall covers not only the broad historical development of American industry, but up-to-the-minute developments in such specific fields as electrical communications, mining, rubber, petroleum, etc. The final two chapters relate the picture of industrial science to national defense, and give a general idea of our outlook for the future. While a great deal of ground is covered, the reader is so interested in the material that he is not unduly alarmed by the variety of subjects. The book is recommended to all Americans, for industrial science is America's greatest achievement—and too few of us understand either how it came into being or why it is important.

TABLES OF CIRCULAR AND HYPERBOLIC SINES AND COSINES FOR RADIAN ARGUMENTS. National Bureau of Standards Applied Mathematics Series 36 (reissue of Mathematical Table 3), 407 pages, 21 × 27 cm. Washington, Government Printing Office, 1953. Price, $3.00.

To meet the continuing demand from scientists and engineers, these tables of circular and hyperbolic sines and cosines are being reissued. These functions are of fundamental importance in pure and applied mathematics.

The interval of the argument and the use of radian measure should prove of particular convenience to the user. Specifically these tables contain sin x, cos x, sinh x and cosh x, $x = 0(.0001)1.9999$, 9D; sinh x and cosh x, $x = 0(.1)10$, 9D; a conversion table of degrees, minutes, seconds ↔ radians and a table of multiples of $\pi/2$.

CALCULATIONS OF ANALYTICAL CHEMISTRY, by Leicester F. Hamilton and Stephen G. Simpson. Fifth edition, 340 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $5.00.

In this fifth edition of a well-known text, much of the material has been rewritten and expanded, particularly in the sections on errors, precision measures, titration curves, electrolytic methods, and potentiometric, conductometric and amperometric titrations. New sections cover iodate and bromate processes and colorimetric methods. This edition continues the emphasis on practical problems in both qualitative and quantitative analysis, and so should serve as a valuable supplement to the regular textbook in chemistry courses.

TABLES OF INTEGRAL TRANSFORMS. Volume I, edited by A. Erdélyi. (Bateman Project Staff.) 391 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.50.

Prepared as a companion to "Higher Transcendental Functions" by the same authors, this is the first of two volumes of

tables of integrals involving higher transcendental functions, and is designed for the use of mathematicians, scientists and engineers. Volume I contains tables of Fourier transforms (sine, cosine, and exponential), of Laplace transforms and inverse Laplace transforms, and of Mellin transforms and inverse Mellin transforms. Many of the integrals tabulated are new, and others, although published, are available only in scattered research papers and hard to obtain periodicals.

FOUR-PLACE TABLES OF TRANSCENDENTAL FUNCTIONS, by W. Flügge. 136 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc.; London, Pergamon Press, Ltd.; 1954. Price, $5.00.

These tables are for use in calculations for which the order of accuracy afforded by the slide rule is sufficient. Containing formulas necessary to handle the functions, the tables deal with trigonometric, hyperbolic, exponential and logarithmic functions, certain of the Bessel functions, and elliptic integrals, the Fresnel integrals, the error function, the sine-, cosine-, and exponential-integral functions, and the gamma function.

The Franklin Institute, a privately endowed institution which exists today only because of the generous benefactions of its supporters, is justly proud of the dividends in human benefits accrued from the investments made through the years by its friends. The accumulation of many bequests for general endowment and income to be used where it will do the most good, is the most important and pressing need of The Franklin Institute today.

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CURRENT TOPICS

Experimental Ionic Pump.—Two General Electric scientists have built a simple experimental air pump with no moving parts, which can produce a vacuum as high as one billionth of normal atmospheric pressure, similar to that in X-ray tubes and TV picture tubes.

Dr. A. M. Gurewitsch and Dr. W. F. Westendorp, of the company's Research Laboratory, said that the device, known as an ionic pump, may eventually simplify exhaustion of radio tubes, and other widely-used vacuum tubes in the process of manufacture.

Pumps presently used for the purpose involve streams of oil or mercury vapor, which are not required in the new device.

In addition, it was said, for some special applications, such a pump might be permanently attached to the tube in which the vacuum must be maintained, and it could easily be used to pump out minute amounts of air of other gases that had leaked in.

One form of the pump consists of a circular stainless steel box, about two inches in diameter and an inch thick. The box is supported between the poles of a powerful Alnico permanent magnet, and is connected by means of a tube to the vessel from which gas is to be exhausted.

Rough exhaustion of air to about one one-hundred-thousandth of an atmosphere, which is easy to attain, is accomplished with an ordinary mechanical pump. Then a valve is opened, to connect the ionic pump with the vessel, and the current is turned on.

Electrical forces break up the gas atoms and hurl the fragments into carbon plates at the sides of the pump, where they stick, so that gas is gradually removed and the desired vacuum is reached.

Explaining the operation, Dr. Westendorp and Dr. Gurewitsch said that inside the box is a tungsten ring, connected to the positively-charged side of the high-voltage circuit which supplies the pump. The metal walls of the box are connected to the negative side of the circuit.

When the current is turned on and the gas pressure has already been reduced well below atmospheric value, an electrical gas discharge occurs. Electrons in this discharge are attracted toward the tungsten ring, but they are prevented from being collected immediately by the field of the magnet and caused to perform numerous oscillations through the ring. During these oscillations, they hit many of the gas atoms, which are partly broken, or "ionized."

The molecular fragments, called "ions," have positive electrical charges, and are pulled toward the pump walls, which are negatively charged. On the way, the ions hit the carbon plates and are driven into them, where they are held. Since they are removed from the open space, gas pressure is reduced.

The scientists said tests show that many cubic centimeters of gas, as measured at atmospheric pressure, can thus be absorbed by the carbon. The absorbed gases can be driven out of the carbon, and the plates made ready to go to work again in absorbing more gas, by simply heating the whole pump.

They said that the ionic pump works on practically all gases, and that it could be used for pumping out all systems where a high vacuum is desired.

Copy Write (*Industrial Bulletin of Arthur D. Little, Inc.*, No. 307).—Carbon copies without carbon paper are now possible. One firm has recently culminated ten years' research on chemically-treated papers with a duplication system capable of turning out four hand-written copies simultaneously or as many as seven with an electric typewriter.

Chemically-treated duplication papers are prepared in two steps. First, a rupturable solid film of gelatin, for instance, containing minute droplets of oil, is coated on the paper base. The oil droplets contain a colorless oil-soluble dye derivative and are released when the film is broken by the typewriter or pencil stroke. Second, a layer of some acidic adsorbent is added, either in the body of the paper, or as a separate layer, perhaps on a separate undersheet. The released oil droplets are adsorbed onto the surfaces of the coating, and a color results. In effect, the oil droplets contain materials that need only this contact with an acid to be changed into a colored state.

In practice, a sheet of paper may be coated on one side with attapulgite clay, an acid material, and on the other with the gelatin film. When a number of sheets are piled up, writing on the top sheet ruptures the gelatin film on the under sides of all the sheets beneath. Oil droplets are transferred onto the clay particles on the top sides of the sheets below, and blue reproductions of the written characters are formed. In another scheme, the clay film is coated directly on the top of the gelatin, with both layers on the top side of the sheet. When pressure is applied, the gelatin film is broken and the oil droplets contact the clay particles immediately above. To date, there is no good indication of the cost of this chemically-treated copy paper, but its makers expect it to compete with carbon paper for forms and sales books. It is not expected to substitute for record carbons of correspondence, memos, etc., in the near future, in view of the probable cost and the necessity for coating letterhead papers.

This new technique of dispensing a liquid chemical where it is needed is suited to many familiar chemical reactions, and may have applications in other industries. Suitable reactions include forming the well-known Prussian blue of a blueprint and production of the purple dyes of the familiar "azo" prints. In any case, the colors formed should be effectively permanent; the new system relies on a long-lived blue dye. "Carbon copy" may therefore become an inadequate term if the new approach becomes widely prevalent.

Moving Sidewalk Being Built.—The world's first commercial moving sidewalk soon will be in operation between the stations of the Erie Railroad and the Hudson and Manhattan Railroad in Jersey City, N. J. It will be capable of handling 10,400 passengers an hour through the interstation tunnel.

The 227-foot conveyor, being built for the Hudson and Manhattan Railroad by the Goodyear Tire and Rubber Co. and the Stephens-Adamson Manufacturing Co., is called a "Speedwalk."

General Electric Company engineers, working in conjunction with Goodyear and Stephens-Adamson, developed the electrical system for the unique application. The equipment includes a fan-cooled motor, a combination reversing starter (fusible type), and a thrustor brake.

The new conveyor will utilize a six-ply rubber and fabric conveyor belt, 5½ feet wide and ⅝ in. thick. It will be driven at a speed of 1½ miles per hour (about half normal walking speed). For 127 feet of its length, the Speedwalk will carry passengers on a 10 per cent grade.

Handrails will move at the same speed as the belt and passengers will be able to step on and off as if it were an escalator. The belt will move in whichever direction the traffic is heavier.

The General Electric reversing starter and totally-enclosed 1200 rpm. fan-cooled motor are both rated at 20 hp., 220-volts, 3-phase, 60 cycles. The thrustor brake is rated at 600 lb. feet continuous and 800 lb. feet intermittent.

Distillation of Sea Water.—Thermocompression distillation of sea water into drinking water at island and desert military bases will be accomplished more rapidly and economically as a result of investigations recently completed by the Engineer Research and Development Laboratories, Fort Belvoir, Va.

The investigations have determined the most effective means of controlling scale formation in equipment. Until now this has been the greatest obstacle to economical distillation. In the thermocompression method, steam from boiling sea water is compressed and recycled to maintain the boiling action. As the latent heat is transferred to sea water the steam condenses to form pure salt free distillate. Scale deposited by the boiling brine on evaporator tube surfaces has in the past doubled the cost of distillation due to lost capacity and efficiency, and the additional labor and materials required.

Frequent descaling with citric acid, which may be accomplished while distillation is in progress, will permit indefinite maintenance of production by field units. A simple procedure has been devised which minimizes the corrosion hazards of acid, time out for cleaning and other interferences.

In addition, the investigation established that descaling may be reduced to a minimum in permanent or semi-permanent installations by means of brine stabilization equipment designed for the predetermined conditions of the location on the basis of essential engineering data. The brine stabilizing method which provides a stabilizer of suspended solids on which the greater part of the scale is deposited has in some instances increased unit production twenty times.

The investigation included laboratory work at the ERDL Sanitary Engineering Branch at Fort Belvoir, Virginia, and field work at the former ERDL Distillation Test Station, Fort Story, Virginia, the Boston Army Base, Boston, Massachusetts, and Kindley Air Force Base, Bermuda. Basic investigations on the equilibria of scale-forming compounds in boiling concentrated sea water were carried out at the University of California. The University also conducted a series of tests at Oakland Army Base, Oakland, California.

Ultrasonics in Medicine.—The use of electronically generated high frequency sound vibrations in the treatment of cancer and in lieu of brain surgery were two topics discussed at the recent IRE convention.

How ultrasonic vibrations have been used in the treatment of cancer in mice was discussed by Earl H. Newcomer of the University of Connecticut who presented a paper on the effects of ultrasound on living cell structure. Although it was reported that half the mice were killed by the treatment because of injury to vital organs caused by the high intensity vibrations, those which survived treatment were characterized by a complete regression of the tumor.

High frequency, high intensity sound has been used in the treatment of mental patients in lieu of surgery according to a report made to the engi-

neers by Dr. P. Lindstrom of the Veterans Administration Hospital of Pittsburgh, who stated that a technique has been developed which will cause minial lesions in the deeper layers of the frontal lobes.

Dr. Lindstrom pointed out that in lobotomies a part of the brain is either cut, removed or otherwise destroyed which can lead to unpredictable hemorrhages and side effects such as epilepsy, loss of bladder control and mental deterioration. Utilizing ultrasound techniques, Dr. Lindstrom continued, most patients improved mentally or obtained pain relief. Minor lesions caused by ultrasound, it was reported, could only be seen in a microscope.

Detection of Films on Electrical Contacts.—The mystery of why certain metal surfaces on wires and contacts fail at times to conduct electricity even though they are physically joined is being solved in many cases with the aid of a tiny platinum wire one-thirtieth as thick as a human hair.

Formed into a loop by scientists at the General Electric Research Laboratory, the thin wire can be used to detect finger prints and other films built up by oxidation or tarnishing. Even invisible films can frequently be detected.

When compressed against a surface, the resilient platinum probe ferrets out films on the most highly buffed and machined surfaces. Measurement of the pressure exerted with a micrometer suggests the type of film present and its electrical strength.

Details about films are studied through measurement of voltage that must be applied through the fine probe in order to break down the film, and by observation of the probing operation through high-powered microscopes.

Developed by Robert H. Savage and Dr. Donald G. Flom, the probe has been used at times to replace difficult chemical means for analyzing such films. Such chemical techniques, it was pointed out, often destroy or damage the surface, while the delicate probe does not affect it.

Savage said that these explorations, though just in the beginning stages, already have detected insulating films on many electrical contacts that otherwise might continue to plague technicians working with them.

The tiny loop recently helped scientists at the laboratory to detect permanent high-resistant areas on silver contacts that had been polished with a high-speed buffer.

They found that the same surfaces, when machined, developed only temporary insulating films which could be removed with solvents.

Phenolic Foam.—A remarkable buoyant plastic material which transforms itself from a liquid resin to an expanded light weight cellular solid state in two or three minutes is being used to replace balsa wood to fill the voids of small carriers. Balsa imparted buoyancy to the ship and prevented listing in the event the hull was damaged by shells. Phenolic foam is also being used for low temperature and acoustical insulations, structural reinforcements for tail sections of airplanes, buoyancy media for boats of all types and for packaging materials. The cost of the liquid resin (unfoamed) is about 50 cents per pound, making this material one of the least expensive foams available. The Navy's first introduction to plastic foams came with the surrender of the German heavy cruiser Prinz Eugen. At the end of the war further knowledge was gained concerning the great military and commercial utilization of vinyl resins in the German war economy. Samples

of the German material were obtained and analyzed in this country. Through cooperation with American industry, the Navy was successful in duplicating and improving on the expanded polyvinyl chloride.

Synthetic Mica for Vacuum Tube Use.—In time of national emergency, mica is probably the most important single strategic material used by the electron tube industry. Because of the difficulty in obtaining adequate quantities of high grade mica during the war, the Bureau of Ships, the Office of Naval Research and the Signal Corps began investigating the possibilities of assuring a domestic supply of electrical grade mica by a synthetic process. One of the most promising materials developed is a hot pressed synthetic mica with a composition similar to natural mica except that fluorine is substituted for the hydroxyl radical commonly found in the natural mineral. The chemicals used in the synthesis are readily available and thus the final product is not strategic. Because of the significance of its use as spacers and output windows in electron tubes, a study of the important characteristics of synthetic mica was undertaken by the Naval Research Laboratory. The mica used was the normal fluor phlogopite which is one of several synthetic micas available. It was selected because it seemed most promising for vacuum tests. From the tests it is believed that this material will play a useful part in vacuum tube applications.

Color X-Ray Photographs.—Medical science now has an advanced tool for studying the human body. It is a system for taking color X-ray photographs, and was reported to the IRE convention by Professor R. Stuart Mackay of the University of California. Experiments show more detail with the color technique than with the black and grey variations in ordinary X-rays. Principles of X-ray color film and direct-viewing color fluoroscopy have been worked out. Presentation on a color TV set has even been considered!

The use of color X-rays is based on the fact that different materials absorb X-rays differently. By taking three ordinary X-ray pictures of a part of the body, each time using a different wavelength or energy, and then combining these pictures by projecting them through different colored filters, a single multicolored image or photographic print results. More technically speaking, the differences in X-ray absorption characteristics are converted into visible colors.

Professor Mackay also described a related process for locating a given element in a body. Here, one alternately observes the body by frequencies slightly higher and lower than the characteristic absorption. The difference in intensity is essentially a measure of the one element alone. A typical application would be the use of an iodine tracer inserted in the body to map out a particular organ. In ordinary X-ray pictures, the blackening from the tracer might not be noticeably different from that of the surrounding body, but the color pictures would show it up.

Oceanographic Observatory.—Under contract to the Navy, the Woods Hole Institute is conducting oceanographic research on the middle of the Atlantic Ocean. Thus far most oceanographic research has been confined to regions of the ocean within a few hundred miles of the continental coasts, but little has been done in the middle of the oceans. What is needed is a ship to be used as an oceanographic laboratory, but a ship is considered too

expensive. Radio transmitter buoys have been built which are being launched from ideally located Bermuda. The buoy makes automatic measurements, and by means of a small automatic transmitter sends measurements back to the Bermuda Observatory every three hours. The problem under investigation at present is the study of surface currents in the Central North Atlantic Ocean.

Nuclear Power Industry.—To establish a nuclear power industry in this country will be a great achievement. If power becomes cheaper and more plentiful, our material standard of living will be raised. In other countries the effect may be even greater. By the accident of history the first use of this great new discovery has been in the development of weapons of war, weapons of appalling magnitude. The nations of the world have today the means to destroy each other. They also have, in this same nuclear energy, a new resource which could be used to lift the heavy burdens of hunger and poverty that keep masses of men in bondage to ignorance and fear. Toward this peaceful development of nuclear power we have, all of us, a high obligation to work with all the ingenuity and purpose we possess.

This is the "Conclusion" of an address by Henry D. Smyth, member of the U. S. Atomic Energy Commission, delivered at the National Meeting of the American Institute of Chemical Engineers, in Washington, March 9, 1954.

New Drug to Treat Burns.—Increased survival rate of severely-burned persons may be realized if results of animal research with a new drug at the University of Wisconsin can be obtained in human patients. UW scientists have shown that the drug, commonly called piperazine, will combat burn shock in animals and increase the rate of survival up to 100 per cent.

Steven E. Jordan, Wisconsin Alumni Research Foundation research assistant, spoke to the American Society for Pharmacology and Experimental Therapeutics, and described research conducted by the departments of anesthesiology, pharmacology, and pharmacy. Jordan said the survival rate for animals suffering from burn shock, but not receiving the drug, ranged from 35 to 45 per cent. One hundred per cent survival was obtained when large amounts of the drug were given following burns. The survival rate for animals receiving smaller amounts of the drug ranged from 82 to 100 per cent. The researcher also pointed out that significant increase in survival was obtained when the drug was given prior to burn. In all experiments the effective dose was only one-half the amount required to kill the animal.

The drug has not yet been tried on humans suffering from burn shock. Normal treatment today is administration of whole blood, plasma, and such drugs as ACTH and cortisone. But even with this treatment many persons die from burn shock.

Action of the drug, which is related to some of the local anesthetics, is not yet known and UW scientists are currently working to determine this.

Research with the drug has been going on at the university for about four years. An earlier report on studies with dogs indicated that piperazine increased survival of animals suffering from shock due to loss of blood.

INDEX TO VOLUME 257

January to June, 1954

References with (N) are Notes from the National Bureau of Standards and those with (C) are Current Topics.

A

Abrasives. Stone saw adapted to cut glass melamine laminates, 262(C)

Acoustics. New underwater acoustic lab in operation, 84(C)

Air cleaning. Electrostatic air cleaner, 262(C)
Precipitron features push-button wash cycle, 87(C)

Air pumps. Experimental ionic pump, 527(C)

Airy stress function in curvilinear coordinates with application to the uniform flexure of a naturally curved spiral beam (Pister), 25

Albrecht, H. O. and C. E. Mandeville: The storage of energy in some activated alkali halide phosphors, 353

Aldehydes. Determination of aldehyde in glucose, 136(N)

Alfvén, H. On H. Alfvén's theory of the effect of magnetic storms on cosmic ray intensity (Swann), 191

Ampère, André-Marie. Interpretation of the Ampère experiments (Moon and Spencer), 203

Ampère force. The Coulomb force and the Ampère force (Moon and Spencer), 305
A new electrodynamics (Moon and Spencer), 369

Analog computers. A versatile electronic function generator (Tomovich), 109

Applied mechanics. Symmetrically loaded circular plates (Stippes and Beckett), 465

Armstrong, Edwin H.: Study to aid courts in cases involving scientific questions, 156(C)

Arresters. New station-type lightning arrester, 316(C)

Artificial mouth. Dental technician develops artificial mouth, 84(C)

Astronomy. Two radio "nebulae" discovered, 274(C)

Atomic energy. Experimental irradiation of foods, 426(C)
First direct conversion of nuclear energy to usable electricity, 345(C)
Giant sphere for atomic sub engine completed, 228(C)
Nuclear power industry, 532(C)
Use for atomic reactor heat waste, 85(C)

Atomic submarine. Giant sphere for atomic sub engine completed, 228(C)

B

Ballistics. Rapid estimation method for long-pointed projectile trajectories (Powley), 221

Beams. The Airy stress function in curvilinear coordinates with application to the uniform flexure of a naturally curved spiral beam (Pister), 25
Some formulas for large deflections of beam columns (Saelman), 125

Beauty, the ideal—of the arts and the sciences (Swann), 431

Beckett, R. E. and M. Stippes: Symmetrically loaded circular plates, 465

Bending. The Airy stress function in curvilinear coordinates with application to the uniform flexure of a naturally curved spiral beam (Pister), 25

Bernoulli's method. A neglected method for resolution of polynomial equations (Taylor), 459

Bioengineering. Vibration machine to explore man's reactions to jiggling, 228(C)

Book reviews
Abbett, Robert W.: Engineering contracts and specifications, 525
Bailar, John C., Jr.: Inorganic syntheses, Vol. IV, 425
Bakal, Carl: Filter manual, 80
Baker, H. Dean, E. A. Ryder and N. H. Baker: Temperature measurement in engineering, Vol. I, 151
Beck, A. H. W.: Thermionic valves, 252

Beckerley, James G., ed.: Annual review of nuclear science, Vol. 3, 337
Bellman, Richard: Stability theory of differential equations, 336
Bikerman, J. J., *et al.*: Foams: theory and industrial applications, 522
Brimley, R. C. and R. C. Barrett: Practical chromatography, 337
Bullen, K. E.: An introduction to the theory of seismology, 340
Burk, Dean, ed.: Cell chemistry, 515
Burlew, John S., ed.: Algal culture: from laboratory to pilot plant, 147
Burr, Irving W.: Engineering statistics and quality control, 523
Byers, Horace R., ed.: Thunderstorm electricity, 72
Conant, James Bryant, ed.: Pasteur's and Tyndall's studies of spontaneous generation, 335
Cook, Arthur L. and Clifford C. Carr: Elements of electrical engineering, 344
de Broglie, ed.: Les applications de la mécanique ondulatoire a l'étude de la structure des molécules, 516
Dodd, R. E. and P. L. Robinson: Experimental inorganic chemistry, a guide to laboratory practice, 522
Draper, C. S., W. McKay and S. Lees: Instrument engineering, Vol. II, 148
Duhem, Pierre: The aim and structure of physical theory, 341
Emmett, Paul H., ed.: Catalysis, Vol. I. Fundamental principles (part I), 519
Erdélyi, A., ed.: Higher transcendental functions, 150
Tables of integral transforms, Vol. I, 525
Evans, Ralph M., W. T. Hanson, Jr. and W. Lyle Brewer: Principles of color photography, 340
Flügge, W.: Four-place tables of transcendental functions, 526
Fourth symposium (international) on combustion, 521
Frisch, O., ed.: Progress in nuclear physics, 517
Fryer, H. C.: Elements of statistics, 417
Gamow, George: Mr. Tompkins learns the facts of life, 420
Garrett, C. G. B.: Magnetic cooling, 511
Gatland, Kenneth W. and Anthony M. Kunesch: Space travel, 151
Getchell, Robert Ward: Qrganic chemistry: a brief course, 425
Giffen, E. and A. Muraszew: The atomisation of liquid fuels, 339
Goldman, Stanford: Information theory, 77
Goody, R. M.: The physics of the stratosphere, 255
Gordy, Walter, William V. Smith and Ralph Trambarulo: Microwave spectroscopy, 149
Gray, Truman S.: Applied electronics, 425
Hadlow, Leonard: Climate, vegetation & man, 79
Hall, Courtney Robert: History of American industrial science, 525
Hamilton, Leicester F. and Stephen G. Simpson: Calculations of analytical chemistry, 525
Hartung, E. J.: The screen projection of chemical experiments, 524
Hausen, Morris H., William N. Hurwitz and William G. Madow: Sample survey methods and theory (two volumes), 71
Heisenberg, Werner, ed.: Kosmische strahlung, 145
Heisenberg, Werner: Nuclear physics, 149
Henderson, J. G.: Metallurgical dictionary, 79
Hinkle, Rolland T.: Kinematics of machines, 79
Hood, Peter: How the earth is made, 517
Hotchkiss, A. G. and H. M. Webber: Protective atmospheres, 256
Jackson, Willis, ed.: Communication theory, 72
Kellogg, F. H.: Construction methods and machinery, 418
Kells, Lyman M.: Elementary differential equations, 344
Kuiper, Gerard P.; ed.: The sun. Vol. I, the solar system, 421
Kyle, P. E.: The closed die forging process, 424
Langer, Rudolph E.: A first course in ordinary differential equations, 344
Laurson, Philip G. and William J. Cox: Mechanics of materials, 423
Lederer, Edgar and Michael Lederer: Chromatography, 335
Leonard, Jonathan Norton: Flight into space, 77
Loew, E. A.: Direct and alternating currents, theory and machinery, 424

Lothers, John E.: Design in structural steel, 253
McFarland, Marvin W., ed.: The papers of Wilbur and Orville Wright, 516
McLachlan, N. W.: Complex variable theory & transform calculus, 422
McShane, E. J., J. L. Kelly and F. V. Reno: Exterior ballistics, 76
Miller, Kenneth S.: Partial differential equations in engineering problems, 79
Morgan, Joseph: Introduction to geometrical and physical optics, 153
Morris, Joe L.: Welding processes and procedures, 424
Morse, Philip M. and Herman Feshbach: Methods of theoretical physics, Vols. I and II, 333
Muskhelishvili, N. I.: Singular integral equations, 520
Newell, Homer E., Jr.: High altitude rocket research, 251
Newsom, Carroll V. and Howard Eves: Introduction to college mathematics, 344
Nixon, Floyd E.: Principles of automatic controls, 147
Pars, L. A.: Introduction to dynamics, 515
Peck, Edson Ruther: Electricity & magnetism, 338
Pennak, Robert W.: Fresh-water invertebrates of the United States, 79
Peterson, R. E.: Stress concentration design factors, 338
Price, Charles C., ed.: Organic syntheses, Vol. 33, 80
Proceedings of the Western Computer Conference, 339
Ramsey, Norman F.: Nuclear moments, 419
Rauscher, Manfred: Introduction to aeronautical dynamics, 145
Read, W. T., Jr.: Dislocations in crystals, 251
Reed, Henry R. and Carl M. Russell: Ultra high frequency propagation, 73
Rees, Paul K. and Fred W. Sparks: College algebra, 424
Review of textile progress, Vol. 4, 423
Richardson, E. G., ed.: Technical aspects of sound, 420
Roe, Harry Burgess and Quincy Claude Ayres: Engineering for agricultural drainage, 424
Rogers, J. S.: Physics for medical students, 518
Rogers, Walter E.: Introduction to electric fields, 520
Rossini, F. D., B. J. Mair and A. J. Streiff: Hydrocarbons from petroleum, 513
Rothe, Frederick S.: An introduction to power system analysis, 150
Salvadori, Mario G. and Ralph J. Schwarz: Differential equations in engineering problems, 514
Scientific American reader, 154
Scientific papers presented to Max Born, 417
Segrè, E., ed.: Experimental nuclear physics, Vol. II, 75
Seybolt, A. U. and J. E. Burke: Procedures in experimental metallurgy, 152
Shaw, F. S.: An introduction to relaxation methods, 255
Shea, Richard F., ed.: Principles of transistor circuits, 75
Smith, Lloyd P.: Mathematical methods for scientists and engineers, 253
Stoever, Herman J.: Essentials of engineering thermodynamics, 421
Sturley, K. R.: Radio receiver design, Part I, 424
Sully, A. H.: Chromium—metallurgy of the rarer metals, No. 1, 512
Tables of circular and hyperbolic sines and cosines for radian arguments, 525
Thielman, Henry P.: Theory of functions of real variables, 73
Timoshenko, Stephen P.: Collected papers, 511
Trathen, Roland H.: Statics and strength of materials, 419
Van Peursem, Ralph L. and Homer C. Imes: Elementary quantitative analysis, 80
Vennard, John K.: Elementary fluid mechanics, 425
von Braun, Wernher, Fred L. Whipple and Willy Ley: Conquest of the moon, 74
White, Marsh, W. and Kenneth V. Manning: Experimental college physics, 344
Wild, F.: Estimation of organic compounds, 78
Wilson, A. H.: Theory of metals, 334
Woodward, P. M.: Probability and information theory, with applications to radar, 418
Young, James F.: Materials and processes, 343

Boundary value. On the flow behind an axially-symmetrical attached curved shock (Pai), 383
Mixed boundary value problems in potential theory (Huth), 121
Buland, Robert N.: Analysis of nonlinear servos by phase-plane-delta method, 37
Burglar alarm. Ultra-sonic burglar alarm, 158(C)
Burnishing wheel. Fibre-glass burnishing wheel, 85(C)

C

Cabinets. "See-Thru" drawer cabinets, 348(C)
Calibration. Screw-thread pitch measurement, 492(N)
Campbell, William S.: Engineering department modernization, 349
Cancer. Chest X-rays for detection of lung cancer, 274(C)
Ultrasonics in medicine, 529(C)
Capacitors. New 50-amp., heavy-duty feed-thru capacitor, 88(C)
Caries. Dental technician develops artificial mouth, 84(C)
Cellulose. Determination of aldehyde in glucose, 136(N)
Chelates. Vermiculite carrier solves chelate application problems, 316(C)
Chemistry. Determination of aldehyde in glucose, 136(N)
Gum turpentine may become a practical source of peroxides, 260(C)
Precise determination of orthophosphate, 231(N)
Chloromycetin effectively combats scrub typhus, 160(C)
Circuitry. Analysis of network response to periodic waves (Seal), 13
The synthesis of sequential switching circuits (Huffman), 161, 275
Communications. First "surface-barrier" transistor, 155(C)
New high-speed facsimile equipment developed, 86(C)
Computers. FOSDIC—a film optical sensing device for input to computers, 399(N)
Metal ultrasonic delay lines, 317(N)
New magnetic drum differential analyzer, 348(C)
New thermal printer, 157(C)
A versatile electronic function generator (Tomovich), 109
Conductivity. Detection of films on electrical contacts, 530(C)
On H. Alfvén's theory of the effect of magnetic storms on cosmic ray intensity (Swann), 191
Semiconducting intermetallic compounds, 49(N)
Conn, Willi M.: Recent progress in solar furnaces for high-temperature research and development work, 1
Control circuits. The synthesis of sequential switching circuits (Huffman), 161, 275
Control devices. New devices to disassemble steam control valves on turbines, 262(C)
Conveyers. Conveyer belt sorter, 133(N)
Moving sidewalk being built, 528(C)
Corrosion. Laboratory measurement of corrosion in soils, 52(N)
Plastic screens being tested, 347(C)
Cosmic rays. On H. Alfvén's theory of the effect of magnetic storms on cosmic ray intensity (Swann), 191
Cotton. USDA cotton-flameproofing treatment, 347(C)
Coulomb force and the Ampère force (Moon and Spencer), 305
Court cases. Study to aid courts in cases involving scientific questions, 156(C)
Cranberries. New use for cranberries, 84(C)
Crime. Science and the peace officer (Hoover), 263
Crystallography. Perfect crystals of pure iron, 429(C)

D

Delay lines. Metal ultrasonic delay lines, 317(N)
Dental technician develops artificial mouth, 84(C)
Differential analyzers. New magnetic drum differential analyzer, 348(C)
A versatile electronic function generator (Tomovich), 109
Differential equations. Analysis of nonlinear servos by phase-plane-delta method (Buland), 37
Distillation of sea water, 529(C)
Drafting. Engineering department modernization (Campbell), 349
Duplication systems. Copy write, 528(C)

E

Elasticity. The Airy stress function in curvilinear coordinates with application to the uniform flexure of a naturally curved spiral beam (Pister), 25
Electric eye. Germanium photocell, 88(C)
Electricity. Detection of films on electrical contacts, 530(C)
First direct conversion of nuclear energy to usable electricity, 345(C)
Interpretation of the Ampère experiments (Moon and Spencer), 203
Electricity and magnetism. A new electrodynamics (Moon and Spencer), 369
Electrodynamics. The Coulomb force and the Ampère force (Moon and Spencer), 305
Interpretation of the Ampère experiments (Moon and Spencer), 203
A new electrodynamics (Moon and Spencer), 369
Electromagnetics. The Coulomb force and the Ampère force (Moon and Spencer), 305
Electromechanics. Conveyer belt sorter, 133(N)
Electron cyclotrons. Microtrons (electron cyclotrons) for X and K band operation (Kaiser), 89
Electron microscope. Pictures of polio virus, 87(C)
Electronics. Electronic translation, 257(C)
First "surface-barrier" transistor, 155(C)
FOSDIC—a film optical sensing device for input to computers, 399(N)
Metal ultrasonic delay lines, 317(N)
NBS electro-optical image processing system, 494(N)
NBS parachute telemetering system, 229(N)
Project "Tinkertoy" drawings available, 346(C)
Semiconducting intermetallic compounds, 49(N)
A versatile electronic function generator (Tomovich), 109
Electrostatics. Electrostatic air cleaner, 262(C)
Miniature electrostatic generator, 135(N)
Precipitron features push-button wash cycle, 87(C)
Emission. The storage of energy in some activated alkali halide phosphors (Albrecht and Mandeville), 353
Engineering department modernization (Campbell), 349
Exterior ballistics. Rapid estimation method for long-pointed projectile trajectories (Powley), 221

F

Facsimile equipment. New high-speed facsimile equipment developed, 86(C)
Federal Bureau of Investigation. Science and the peace officer (Hoover), 263
Fibre-glass burnishing wheel, 85(C)
Film. New industrial X-ray film, 82(C)
Flameproofing. USDA cotton-flameproofing treatment, 347(C)
Flexure. The Airy stress function in curvilinear coordinates with application to the uniform flexure of a naturally curved spiral beam (Pister), 25
Fluid mechanics. New equipment for demonstrating fluid mechanics, 202(C)
Fluoroscopy. The image amplifier in movies and TV, 82(C)
Foam. Phenolic foam, 530(C)
Food. Experimental irradiation of foods, 426(C)
Franklin, Benjamin. Plan for the celebration of the 250th anniversary of the birth of Benjamin Franklin, 60
The Franklin Institute
Bartol Research Foundation Notes:
Albrecht, H. O. and C. E. Mandeville: The luminescence of beryllium oxide, 249
Danforth, W. E.: Electrolytic disintegration of thorium oxide, 507
Danforth, W. E.: Ionic conduction in thorium oxide, 247
Mandeville, C. E. and H. O. Albrecht: Phosphorescence of and energy storage in some ultraviolet emitting phosphors, 506
Marshall, J. F., R. A. Shatas and M. A. Pomerantz: Target-thickness dependence of delta-ray emission produced by 1.3 mev electrons, 509
Metzger, F. R. and W. B. Todd: Resonance fluorescence with nuclei: Hg^{198}, 248
Pomerantz, M. A., J. F. Marshall and R. A. Shatas: Forward/backward ratio of secondary emission produced by 1.3 mev electrons, 508
Pomerantz, M. A., R. A. Shatas and J. F. Marshall: Forward-backward

ratio of secondary electrons emitted from thin targets, 250
Saraf, Babulal: The inner Bremsstrahlung in A^{37} and Ge^{71}, 506
Saraf, Babulal: The inner Bremstrahlung for Cs^{131}, 248
Shatas, R. A., J. F. Marshall and M. A. Pomerantz: Energy distribution of secondary electrons produced by 1.3 Mev primary electrons, 250
Shatas, R. A., M. A. Pomerantz and J. F. Marshall: Electron bombardment conductivity of MgO single crystal, 509
Snowdon, S. C. and W. D. Whitehead: Angular distribution of neutrons scattered from cadmium, tin, and bismuth, 249
Swann, C. P.: Large generator work, 509
Swann, C. P. and F. R. Metzger: Inelastic scattering of neutrons by Ba^{137} and Hg^{199}, 510
Swann, W. F. G.: Abnormalities in lead absorption curves, 505
Swann, W. F. G.: Acceleration to cosmic ray energies by betatron action, 505
Swann, W. F. G.: On H. Alfvén's theory of the effect of magnetic storms on cosmic ray intensity, 247
Varma, Jagdish: Nuclear energy levels of Tl^{203}, 247
Varma, Jagdish: Radiations from Cd^{115} (43-days), 507
Biochemical Research Foundation Notes:
Cramer, Francis B.: The alpha form of 2-desoxy-D-glucose, 69
de Courcy, S. J., Jr., J. O. Ely and M. H. Ross: *Abstract of* Effect of ultraviolet light on deoxyribonucleic acid in rat thymocytes, 326
McDonald, Ellice: Seminar on the separation of biochemically important substances by ion-exchange chromatography, 239
McDonald, Ellice: The similarity of cancer glycolysis and yeast fermentation, 411
Ross, M. H.: Sugar discrimination by experimental rats, 500
Ross, M. H. and J. O. Ely: The variation of magnesium requirement for optimal activation of alkaline phosphatase, 143
Committee on Science and the Arts: *Abstract of* Proceedings, February 10, 1954, 234; April 14, 1954, 405; May 12, 1954, 497
Franklin Institute Laboratories Notes:
Glassman, I.: A 98-point thermocouple digital recording system, 501
Good, Robert: Deformation of rotating bands on artillery projectiles, 64
Hay, A. Donald: A design for comfortable working conditions, 414
Myerson, Albert L.: Combustion studies: carbon disulfide-oxygen explosions, 329
Robinson, George: Friction research, 243
Journal of The Franklin Institute, 59, 141, 234, 322, 405, 499
Library, 61, 139, 235, 323, 406, 498
Membership, 56, 142, 234, 324, 406, 497
Museum Notes: Platinum, 67; the human heart, 237; the bicycle collection, 327; "The Rocket"—early steam locomotive, 409; Edison's first lamp, 503
Plan for the celebration of the 250th anniversary of the birth of Benjamin Franklin, 60
Stated meetings: Minutes, December 16, 1953, 55; January 20, 1954 (Annual Meeting), 138; February 17, 1954, 233; March 17, 1954, 322; April 21, 1954, 404; May 19, 1954, 496
Frequency modulation. Nonlinear study of frequency modulated oscillator (Gross), 481
Function generator. A versatile electronic function generator (Tomovich), 109
Function theory. Mixed boundary value problems in potential theory (Huth), 121
Furniture. Engineering department modernization (Campbell), 349

G

Gages. A shaft displacement indicator, 491(N)
Generators. Miniature electrostatic generator, 135(N)
Germanium. Germanium photocell, 88(C)
New refining technique for germanium, 427(C)
Glare. Aerosol spray softens glare in photographing shiny objects, 228(C)

Glass. Stone saw adapted to cut glass melamine laminates, 262(C)
Graphic arts. New thermal printer, 157(C)
Graphical solution. Analysis of nonlinear servos by phase-plane-delta method (Buland), 37
Gross, Josef: Nonlinear study of frequency modulated oscillator, 481
Gum turpentine may become a practical source of peroxides, 260(C)

H

High temperature alloys. First commercial production of vacuum melted high temperature alloys, 304(C)
High-temperature research. Recent progress in solar furnaces for high-temperature research and development work (Conn), 1
Hoover, John Edgar: Science and the peace officer, 263
Huffman, D. A.: The synthesis of sequential switching circuits, 161, 275
Huth, J. H.: Mixed boundary value problems in potential theory, 121

I

Image amplifier in movies and TV, 82(C)
Inspection. Automatic inspection device, 156(C)
New industrial X-ray film, 82(C)
Instruments. Detection of films on electrical contacts, 530(C)
Miniature electrostatic generator, 135(N)
New 50-amp., heavy-duty feed-thru capacitor, 88(C)
New radiation detector, 304(C)
A shaft displacement indicator, 491(N)
Sine-cosine potentiometer, 480(C)
Vacuum ultra-violet spectrometer, 83(C)
Insulation. Automatic inspection device, 156(C)
Phenolic foam, 530(C)
Ionic pump. Experimental ionic pump, 527(C)
Iron. Perfect crystals of pure iron, 429(C)

K

Kaiser, H. F.: Microtrons (electron cyclotrons) for X and K band operation, 89

L

Laboratory equipment. New equipment for demonstrating fluid mechanics, 202(C)
Ladders. A safe, rolling step ladder, 427(C)
Law enforcement. Science and the peace officer (Hoover), 263
Light. Effect of light on coated groundwood papers, 402(N)
Lighting. Degradation effects of museum light sources, 320(N)
Lightning. New station-type lightning arrester, 316(C)
Linguistics. Electronic translation, 257(C)

M

Magnetic drums. New magnetic drum differential analyzer, 348(C)
Magnetic field. The Coulomb force and the Ampère force (Moon and Spencer), 305
Magnetic storms. On H. Alfvén's theory of the effect of magnetic storms on cosmic ray intensity (Swann), 191
Magnetic tape recording of television pictures, 157(C)
Magnetostriction. Ultra-sonic burglar alarm, 158(C)
Mandeville, C. E. and H. O. Albrecht: The storage of energy in some activated alkali halide phosphors, 353
Mathematics. Analysis of network response to periodic waves (Seal), 13
Mixed boundary value problems in potential theory (Huth), 121
A neglected method for resolution of polynomial equations (Taylor), 459
Maxwell's theory. A new electrodynamics (Moon and Spencer), 369
Measurement. Laboratory measurement of corrosion in soils, 52(N)
Precise determination of orthophosphate, 231(N)
Screw-thread pitch measurement, 492(N)
Medicine. Butterscotch-flavored form of penicillin, 428(C)
Chest X-rays for detection of lung cancer, 274(C)
Chloromycetin effectively combats scrub typhus, 160(C)
Color X-ray photographs, 531(C)
The image amplifier in movies and TV, 82(C)
New drug to treat burns, 532(C)
New use for cranberries, 84(C)
Pictures of polio virus, 87(C)
Ultrasonics in medicine, 529(C)
Vibration machine to explore man's reactions to jiggling, 228(C)

Metallurgy. New refining technique for germanium, 427(C)
Perfect crystals of pure iron, 429(C)
Mica. Automatic inspection device, 156(C)
Synthetic mica for vacuum tube use, 531(C)
Military engineering. Distillation of sea water, 529(C)
High speed teleprinter, 81(C)
New high-speed facsimile equipment developed, 85(C)
Phenolic foam, 530(C)
Superior optical cement, 346(C)
Synthetic mica for vacuum tube use, 531(C)
Moon, Parry and Domina Eberle Spencer: The Coulomb force and the Ampère force, 305
Interpretation of the Ampère experiments, 203
A new electrodynamics, 369
Motors. Totally-enclosed unit-cooled d-c. motor, 159(C)
Museums. Degradation effects of museum light sources, 320(N)
Music. Beauty, the ideal—of the arts and the sciences (Swann), 431

N

Nebulae. Two radio "nebulae" discovered, 274(C)
Networks. Analysis of network response to periodic waves (Seal), 13
The synthesis of sequential switching circuits (Huffman), 161, 275
Nonlinear servomechanisms. Analysis of nonlinear servos by phase-plane-delta method (Buland), 37
Nonlinear study of frequency modulated oscillator (Gross), 481
Nuclear physics. First direct conversion of nuclear energy to usable electricity, 345(C)
Microtrons (electron cyclotrons) for X and K band operation (Kaiser), 89
Nuclear power industry, 532(C)
The storage of energy in some activated alkali halide phosphors (Albrecht and Mandeville), 353

O

Oceanography. Oceanographic observatory, 531(C)
Optics. FOSDIC—a film optical sensing device for input to computers, 399(N)
NBS electro-optical image processing system, 494(N)
Superior optical cement, 346(C)
Orthophosphate. Precise determination of orthophosphate, 231(N)
Oscillators. Nonlinear study of frequency modulated oscillator (Gross), 481

P

Pai, S. I.: On the flow behind an axially symmetrical attached curved shock, 383
Paper. Copy write, 528(C)
Effect of light on coated groundwood papers, 402(N)
Parachutes. NBS parachute telemetering system, 229(N)
Particle accelerators. Microtrons (electron cyclotrons) for X and K band operation (Kaiser), 89
Penicillin. Butterscotch-flavored form of penicillin, 428(C)
Periodic wave analysis. Analysis of network response to periodic waves (Seal), 13
Peroxides. Gum turpentine may become a practical source of peroxides, 260(C)
Perturbation theory. On the flow behind an axially symmetrical attached curved shock (Pai), 383
Phase-plane-delta method. Analysis of nonlinear servos by phase-plane-delta method (Buland), 37
Phosphorescence. The storage of energy in some activated alkali halide phosphors (Albrecht and Mandeville), 353
Photocell. Germanium photocell, 88(C)
Photography. Aerosol spray softens glare in photographing shiny objects, 228(C)
Color X-ray photographs, 531(C)
NBS electro-optical image processing system, 494(N)
Piperazine. New drug to treat burns, 532(C)
Pister, Karl S.: The Airy stress function in curvilinear coordinates with application to the uniform flexure of a naturally curved spiral beam, 25
Plastics. Fibre-glass burnishing wheel, 85(C)
Phenolic foam, 530(C)
Plastic screens being tested, 347(C)

"See-Thru" drawer cabinets, 348(C)
Transducer for polymer research, 429(C)
Poliomyelitis. Pictures of polio virus, 87(C)
Polymers. Transducer for polymer research, 429(C)
Polynomial equations. A neglected method for resolution of polynomial equations (Taylor), 459
Potential theory. Mixed boundary value problems in potential theory (Huth), 121
Potentiometer. Sine-cosine potentiometer, 480(C)
Power. Nuclear power industry, 532(C)
Powley, Homer S.: Rapid estimation method for long-pointed projectile trajectories, 221
Printing. High speed teleprinter, 81(C)
New thermal printer, 157(C)
Project Tinkertoy. Project "Tinkertoy" drawings available, 346(C)
Protective coatings. Effect of light on coated groundwood papers, 402(N)
Plastic screens being tested, 347(C)
Pumps. Belt-driven deep well pump, 430(C)
Experimental ionic pump, 527(C)

R

Radioactivity. New radiation detector, 304(C)
Reactors. Use for atomic reactor heat waste, 85(C)
Recording. Magnetic tape recording of television pictures, 157(C)
Reproduction. Copy write, 528(C)
Rotating sectors. Recent progress in solar furnaces for high-temperature research and development work (Conn), 1
Rubber. Moving sidewalk being built, 528(C)

S

Saelman, B.: Some formulas for large deflections of beam columns, 125
Safety engineering. New radiation detector, 304(C)
A safe, rolling step ladder, 427(C)
Science. Beauty, the ideal—of the arts and the sciences (Swann), 431
Science and the peace officer (Hoover), 263
Study to aid courts in cases involving scientific questions, 156(C)
Scintillation counting. The storage of energy in some activated alkali halide phosphors (Albrecht and Mandeville), 353
Scott, John. John Scott medal award, 458(C)
Screens. Plastic screens being tested, 347(C)
Screw threads. Screw-thread pitch measurement, 492(N)
Seal, Philip M.: Analysis of network response to periodic waves, 13
Semiconducting intermetallic compounds, 49(N)
Servomechanisms. Analysis of nonlinear servos by phase-plane-delta method (Buland), 37
Shock. On the flow behind an axially symmetrical attached curved shock (Pai), 383
Sine-cosine potentiometer, 480(C)
Small arms. Rapid estimation method for long-pointed projectile trajectories (Powley), 221
Soils. Laboratory measurement of corrosion in soils, 52(N)
Solar furnaces. Recent progress in solar furnaces for high-temperature research and development work (Conn), 1
Solid state electronics. Semiconducting intermetallic compounds, 49(N)
Sorters. Conveyer belt sorter, 133(N)
Sound. New underwater acoustic lab in operation, 84(C)
Ultrasonics in medicine, 529(C)
Spectrometer. Vacuum ultra-violet spectrometer, 83(C)
Spencer, Domina Eberle and Parry Moon: The Coulomb force and the Ampère force, 305(C)
Interpretation of the Ampère experiments, 203
A new electrodynamics, 369
Sprayers. New tractor sprayer attachment, 202(C)
Sprays. Aerosol spray softens glare in photographing shiny objects, 228(C)
Steady-state heat flow. Mixed boundary value problems in potential theory (Huth), 121
Steady-state inverse transforms. Analysis of network response to periodic waves (Seal), 13
Sterilization. Experimental irradiation of foods, 426(C)

Stippes, M. and R. E. Beckett: Symmetrically loaded circular plates, 465
Storage. "See-Thru" drawer cabinets, 348(C)
Stream function. On the flow behind an axially symmetrical attached curved shock (Pai), 383
Strength of materials. Some formulas for large deflections of beam columns (Saelman), 125
Symmetrically loaded circular plates (Stippes and Beckett), 465
Structural analysis. Some formulas for large deflections of beam columns (Saelman), 125
Swann, W. F. G.: Beauty, the ideal—of the arts and the sciences, 431
On H. Alfvén's theory of the effect of magnetic storms on cosmic ray intensity, 191
Switching circuits. The synthesis of sequential switching circuits (Huffman), 161, 275

T

Taylor, William C., Jr.: A neglected method for resolution of polynomial equations, 459
Teaching aids. New equipment for demonstrating fluid mechanics, 202(C)
Telemetering system. NBS parachute telemetering system, 229(N)
Teleprinter. High speed teleprinter, 81(C)
Television. Magnetic tape recording of television pictures, 157(C)
Testing. Results of seven-year truck research program, 261(C)
Textiles. USDA cotton-flameproofing treatment, 347(C)
Thermal printer. New thermal printer, 157(C)
Tomovich, Rajko: A versatile electronic function generator, 109
Tractors. New tractor sprayer attachment, 202(C)
Trajectory computation. Rapid estimation method for long-pointed projectile trajectories (Powley), 221
Transducer for polymer research, 429(C)
Transistors. First direct conversion of nuclear energy to usable electricity, 345(C)
First "surface-barrier" transistor, 155(C)
Translation. Electronic translation, 257(C)
Transportation. Moving sidewalk being built, 528(C)
Trucks. Hardwood floors for trucks, 480(C)
Results of seven-year truck research program, 261(C)
Turbines. New devices to disassemble steam control valves on turbines, 262(C)
A shaft displacement indicator, 491(N)
Typhus. Chloromycetin effectively combats scrub typhus, 160(C)

U

Ultra high frequency. Microtrons (electron cyclotrons) for X and K band operation (Kaiser), 89
Ultrasonics. Ultra-sonic burglar alarm, 158(C)
Ultrasonics in medicine, 529(C)
Underwater research. New underwater acoustic lab in operation, 84(C)
Ursolic acid. New use for cranberries, 84(C)

V

Vacuum. First commercial production of vacuum melted high temperature alloys, 304(C)
Vacuum ultra-violet spectrometer, 83(C)
Vacuum tubes. Experimental ionic pump, 527(C)
Synthetic mica for vacuum tube use, 531(C)
Valves. New devices to disassemble steam control valves on turbines, 262(C)
Vermiculite carrier solves chelate application problems, 316(C)
Vibration machine to explore man's reactions to jiggling, 228(C)

W

Waste heat. Use for atomic reactor heat waste, 85(C)
Water. Distillation of sea water, 529(C)
Wood. Hardwood floors for trucks, 480(C)

X

X-rays. Chest X-rays for detection of lung cancer, 274(C)
Color X-ray photographs, 531(C)
New industrial X-ray film, 82(C)
The storage of energy in some activated alkali halide phosphors (Albrecht and Mandeville), 353

Z

Zone-melting. New refining technique for germanium, 427(C)